Green roofs for managing storm water runoff

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Outline

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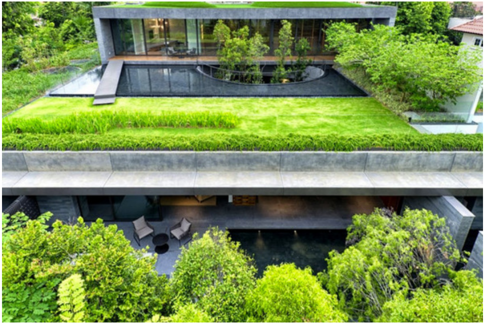
What is a green roof?

A green roof is a roof of a building that is partially covered with vegetation and a growing medium, planted over a waterproofing membrane

(Wikipedia)



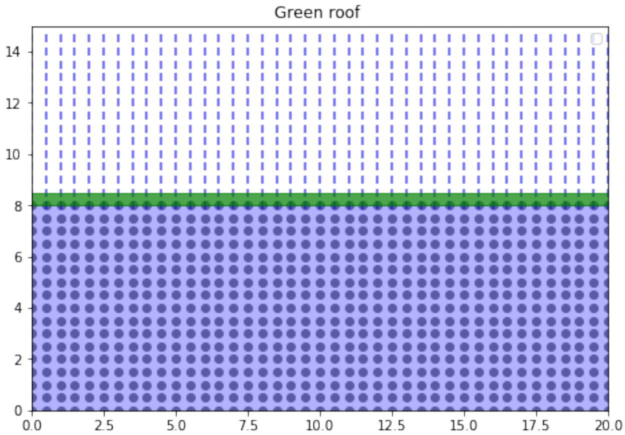
(image credit to domain)

Problem statement

We try to analyze the performance of a green roof in reducing the annual runoff (hydraulic load) from the roof of a building.
Q: What makes the biggest difference; soil depth or soil texture?

Our model

In order to analyze it, we have make a simplified model of a horizontal green roof



To consider

- Porosity
- Permeability
- Void space
- Soil moisture
- Water saturation

We consider our soil as a porous medium.

Outline

Pressure

The over burden pressure P_s in a point(on the liquid) in a porous medium is given by:

$$P_s = P_0 + \text{Pression due to the weight of fluid above}$$

As we have:

$$P = \frac{Weight}{Area}$$

$$w = mg$$

$$\rho = \frac{m}{V} \qquad \Rightarrow m = \rho V$$

$$P = \frac{\rho V g}{A} \qquad \text{but } \frac{V}{A} = z$$

$$P = \rho g z$$

Flow rate

Suppose the rainwater fluid flows through the porous medium (soil) at a volume rate Q. As Darcy's assumptions (

- Single phase flow
- Homogeneous porous medium
- Vertical flow
- non reactive fluid
- Single geometry

)are fulfilled ,the pressure gradient $\overrightarrow{\nabla} P$ is related to the flow rate Q by

$$Q = -\frac{A}{\mu} \bar{\bar{K}} \overrightarrow{\nabla} P$$

Where:

- μ : dynamic viscosity of the fluid
- K: permeability tensor(2nd order and depends on the geometry of a porous medium)

We define $\frac{\overrightarrow{Q}}{A} = \overrightarrow{V_d}$(Darcy's velocity) $\overrightarrow{V_d} = -\frac{\bar{\bar{K}}}{\mu}\overrightarrow{\nabla}P$
But we know that the liquid has a mass so it also make its contribution to the velocity.
So the Darcy's model is given by:
$\overrightarrow{V} = -\frac{\bar{\bar{K}}}{\mu}(\overrightarrow{\nabla}P - \rho\overrightarrow{g})$

Constant velocity

If the porous medium is isotropic, the matrix $\bar{\bar{K}}$ is constant scalar k.
And as we consider only the vertically direction we assume that the velocity is one dimensional.
And $\overrightarrow{\nabla} P$ collapse just to $\frac{\partial P}{\partial z}$
So we end up with
$V = -\frac{k}{\mu}\left(\frac{\partial P}{\partial z} - \rho g\right)$

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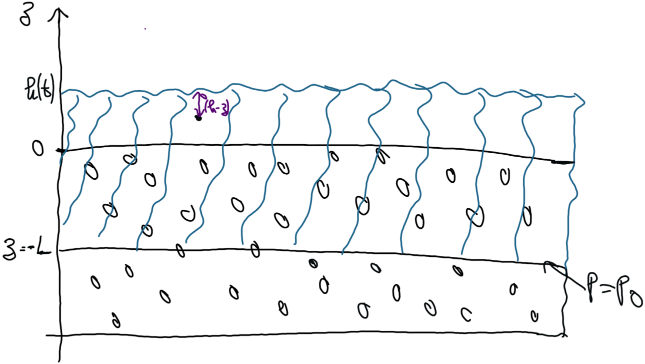
Water depletion and time

In this section we are interested in understanding the water depletion in soil as time goes.



Water depletion and time

- The pressure distribution in the porous medium (soil) satisfies:
$$\frac{\partial^2 p}{\partial z^2} = 0$$
- Therefore:
$P(z, t) = Az + B$
To solve this equation we need boundaries conditions
- The pressure in the water is given by:
$$P_{wat} = P_0 + \rho g(h - z)$$
where $0 < z < h$

Water depletion and time

- Using the boundaries conditions: $z = 0, P(0,t) = P_0 + \rho g h$ and $z = -L, P = P_0$ we have:
  $P = P_0 + \rho g h \left(1 + \frac{z}{L}\right)$
- In the porous medium the Darcy's Law applies:

$$\frac{\mu v}{k} = \frac{\partial P}{\partial z} - \rho g$$

- Using the previous equation and knowing that $v = \frac{dh}{dt}$ and $h_0 = h + \phi L$:

$$\frac{dh}{dt} = -\frac{k}{\mu}\rho g \left[\frac{h_0 + h(\phi - 1)}{h_0 - h}\right]$$

Process for finding time for water depletion

- Solving the previous equation leads to:

$$t = \frac{\mu}{\rho g k(1-\phi)} \left[(h_0 - h) + (h_0 + \frac{h_0}{(\phi - 1)}) ln \left(\frac{h_0 + (\phi - 1)h}{\phi h_0} \right) \right]$$

- We then set: $\bar{t} = \frac{\rho g k}{\mu h_0} t$ and $\bar{h} = \frac{h}{h_0}$ to have a dimensionless expression for t
- Therefore:

$$\bar{t} = \frac{1}{1-\phi} \left[(1 - \bar{h}) - \frac{\phi}{1-\phi} ln \left(1 + \frac{(1-\phi)()1 - \bar{h}}{\phi} \right) \right]$$

Plot of water depletion with time

For different values of ϕ we got the following:

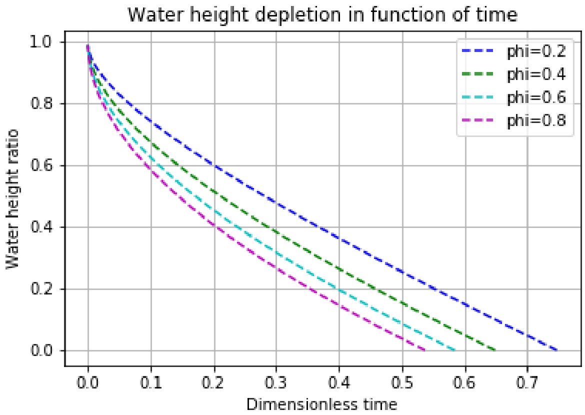


Figure: Water height depletion in function of time

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Conclusion

In this work, given our model and our assumptions, we found that:

- the water velocity through the medium is constant
- the water deplete in time t, which depends on soil parameters such as porosity, permeability.

Conclusion

Besides, we explore other alternatives which for now didn't lead us to significant results. We considered:

- equating the volume of water depleting in the porous medium (with the volume of runoff) to the volume of rainfall (we failed modelling the rainfall in function of time).
- the balance law which involves the soil saturation, the water flux and the rainfall.