AIR TRAFFIC FLOW MANAGEMENT REROUTING PROBLEM (ATFMRP)

A.S. Alockukwu*, F. Getachew †, I. Campbell‡,
A. Adewumi§ and M.M. Ali¶

Industry Representative

A.S. Alochukwu

Study group participants

A. Mgudlwa, A. Nanyanzi, A. Tajudeen, F. Getachew, S. Omer,
T. Zeratsion, T. Lebese, X. Vukeya, I. Campbell, A. Adewumi and M.M. Ali.

Abstract

Air traffic and the air transportation industry have grown rapidly in recent times and this has resulted in increased demand for airport and airspace resources. As a result, flight delays and other congestion problems have resulted, especially at peak travel times and due to poor weather conditions and other unforeseen factors. Consequently, air traffic management has become more complex and efficiently managing air traffic flow more difficult. In this work the Air Traffic Flow Management Problem (ATFM) is described, and the related Air Traffic Flow Management Rerouting Problem (ATFMRP) addressed. The ATFMRP deals with the modelling and optimisation of air traffic flow after disruptions. A neighbourhood search algorithm is proposed to locally re-optimise a schedule after a disruption.

*School of Mathematics, Statistics and Computer Science, University of KwaZulu-Natal, Durban, South Africa, *email: alex.somto@gmail.com* or *Adewumia@ukzn.ac.za*

†African Institute for Mathematical Sciences-South Africa (AIMS-SA), 5-8 Melrose Road, Muizenberg, Cape Town, South Africa, *email: frether@aims.ac.za*

‡School of Mechanical, Industrial & Aeronautical Engineering, University of the Witwatersrand, Johannesburg, South Africa, *email: ian.campbell@wits.ac.za*

§School of Mathematics, Statistics and Computer Science, University of KwaZulu-Natal, Durban, South Africa, *email: adewumi@ukzu.ac.za*

¶School of Computer Science and Applied Mathematics, University of the Witwatersrand, Private Bag 3, WITS 2050, Johannesburg, South Africa, *email: montaz.ali@wits.ac.za*

1 Introduction

The Air Traffic Flow Management Problem (ATFMP) is designed to model and optimise air traffic over a defined geographical area, and without exceeding airport or route capacity. It is a planning activity designed to address overcapacity problems which occur either when airspace capacity is reduced or when demand is high. The ATFMP manages air traffic to ensure safe and efficient flow of aircraft throughout the airspace at the lowest cost.

The ATFMP prevents over capacity of airports and flight routes by modifying the departure times and trajectories of flights, either by assigning ground holding delay, airborne holding delay or various other control actions, including rerouting of flights, flight cancellation, speed control, etc.

A fundamental challenge for air traffic management arises when there is a system disruption, due to weather conditions, equipment outages or air traffic demand surges. These disruptions can be unpredictable and cause significant capacity problems. For instance, there are temporary and substantial reductions in airspace and airport capacity whenever there are adverse weather conditions.

Air traffic managers are faced with challenges when the number of flights departing or arriving from a certain airport as well as the number of aircraft traversing a particular sector of airspace exceeds the usual capacity. This may be as a result of a change in the number of runways available, air traffic control (ATC) capacity, airspace restrictions and restrictions as to which aircraft can follow an aircraft of a given class. In such cases, the air traffic managers must find optimal scheduling strategies that mitigate congestion as well as minimise delay costs.

Since disruption will always occur at some time, the question that arises is how to formulate the problem to account for these uncertainties in the system, as well as how to re-optimise the schedule after disruptions using rerouting or departure delay options (ATFMRP). The ATFM can be large and hence difficult to solve in terms of CPU time, hence a computationally efficient model is needed for the ATFMRP.

The problem was investigated as part of the activities of the Mathematics in Industry Study Group (MISG) in January 2016, South Africa. The assigned group investigated methods of formulating and solving the ATFMRP. Artificially constructed data sets were used.

2 Problem statement and objectives

The ATFMP can be described as follows:
Given an airspace system, consisting of a set of airports, airways, and sectors, each with its own capacity for each time period, t, over a time horizon of T periods, and given flight schedules through the airspace system during T, we want to find good and optimal scheduling ATFM strategies that not only mitigate congestion problems but also minimize delay costs while satisfying the airport and en-route airspace capacity constraints.

The proposed model must provide the amount of ground and airborne delay to be assigned to flights such that all capacity constraints are satisfied, while minimizing a function including the associated cost of the total delay, and taking into consideration all other possible control actions for ATFM, for example, re-routing.

3 Aircraft traffic management

Civilian airspace is designated controlled or uncontrolled. Uncontrolled airspace includes some low altitudes, remote regions and low-traffic areas. Controlled air traffic is managed by air traffic control (ATC) via ATC centres over a predefined geographic area. Air Route Traffic Control Centres (ARTCCs) control traffic between ATCs. In high traffic density areas, such as parts of Europe and USA, an ATFM system is implemented to control flow on an aggregate scale [**?**].

The geographical control area is split into low-level en-route sectors, which may or may not have airports within them. Capacity constraints for sectors and airports are defined by the Operationally Acceptable Level of Traffic (OALT). Sector capacity is defined by a maximum number of aircraft allowed in a specific sector, normally from 10 to 20. The low-level ARTCC sectors for the USA are shown in Figure 1.

Figure 1: Low-Altitude ARTCC Sectors for the USA [6].

At the airport, the OALT is defined by Airport Acceptance Rates (AARs). The AAR for an airport is an hourly forecast of arrival capacity for an airport. Arrival capacity depends on many factors (for example, aircraft type due to wake induced vortices) and weather conditions. On a regional scale, a traffic Management Unit (TMU) coordinates traffic across sectors within a centre, and across neighbouring centres. A Central Flow Management Unit (CFMU) might control flows on a national or quasi-national level. In the USA, this is the ATCSCC and in Europe EUROCONTROL in Brussels [**?**].

Each airline will provide a schedule, designed to meet the demand it is expecting. The ARTCCs must then route the aircraft through the sectors to their destinations. Various efforts have been made to optimise these overall aircraft schedules, such that sector capacities and airport take-off and landing capacities are not exceeded, and aircraft reach their destinations at the lowest possible cost, normally meaning the shortest, most direct route. However, aircraft may need to be rerouted on longer routes if sector capacities are exceeded [**?**].

Models have been presented by Bertsimas and Patterson [**?**], and by Bertsimas et al.[**?**] for the European environment. The model by Bertsimas and Patterson is an NP-hard binary integer programming problem, formulated for a multi-commodity network flow model with side constraints.

Data required includes:

- Set of flights, set of airports, set of time periods, set of pairs of flights that are continued.
- Number of sectors in the path of a flight.
- Departure capacity of airports at a time period.
- Arrival capacity of airports at time periods.
- Scheduled departure and arrival times.
- Aircraft turnaround times (TATs).
- Cost of holding aircraft on the ground and in the air for one time period.
- Number of time periods that flights spend in a sector.

4 ATFMP formulation

4.1 Air space

As an example, Figure 2 shows a six sector geographical region with four airports in four of the sectors, sectors A, D, E and F. Each flight passes through various sectors while en-route to its destination. The air space map in Figure 2 can be represented in graph form, shown in Figure 2, with sectors as nodes and routes between sectors as arcs. The arcs represent possible routes from one sector to another. Each arc has a capacity constraint and a travel cost. The aircraft flying time can be used as the arc cost. For this example, sector capacities (arc capacities) are all the same, and are set at 3.

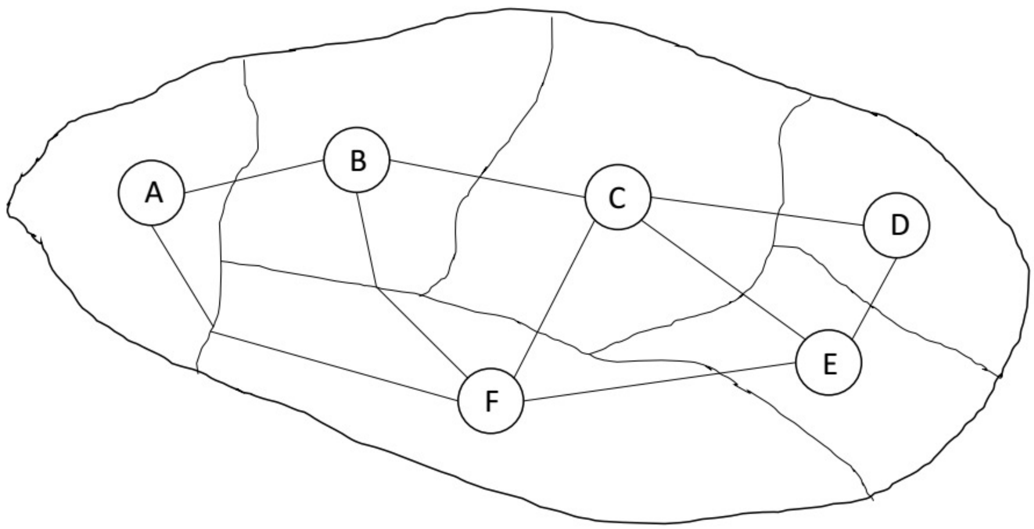

Figure 2: Map of the Air Space and Associated Network.

4.2 Time-space network

A solution to an ATFM problem can naturally be represented by a time-space network, where the nodes represent a sector at a specific time, and the arcs represent either a flight leg or a ground waiting time. As an example, Table 1 shows a schedule of flights involving the six sectors in Figure 2. The table includes expected time of departure (ETD), expected time of arrival (ETD), as well as the route the aircraft will take, shown as the sectors through which it will pass.

Note that this example schedule is infeasible, since certain arc capacities would be exceeded. However, it would be difficult and time-consuming to create manual schedules which are feasible.

Table 1: Example Flight Schedule

Flight	ETD	Origin	ETA	Destination	Sectors
1	06:30	A	08:35	D	ABCD
2	06:30	A	08:40	E	AFE
3	07:00	F	08:35	D	FCD
4	07:00	A	08:05	E	ABCE
5	07:00	D	09:00	F	DECF
6	07:30	D	09:10	E	DCE
7	07:00	D	09:35	A	DEFA
8	07:30	F	09:05	D	FCD
9	07:15	D	09:20	A	DCBA
10	07:45	D	09:20	F	DCF
11	07:30	A	09:35	D	ABCD
12	06:45	F	09:10	E	FBCE
13	07:15	A	09:25	E	AFE
14	08:00	D	09:25	F	DEF
15	07:15	F	09:15	A	FCBA
16	07:15	E	09:20	A	ECBA

A time-space network for the first 5 flights for this example is shown in Figure 3, with flight 1 in black, flight 2 in red, flight 3 in green, flight 4 in blue and flight 5 in purple. The time axis (horizontal axis) is divided into discrete time periods (15 minutes for this case), and the sectors (A - F) are shown on the vertical, or space, axis.

4.3 Mathematical formulation

The ATFMP can be formulated as a multi-commodity network flow problem over a time-space network using binary integer variables x_{fis}, representing whether a particular flight is used or not. Index f represents the schedule flight number and index i represents the route the flight will take. For example, a flight from A to D in Figure 2 could fly route A-B-C-D, A-F-E-D, A-B-F-E-D, etc., so each of these

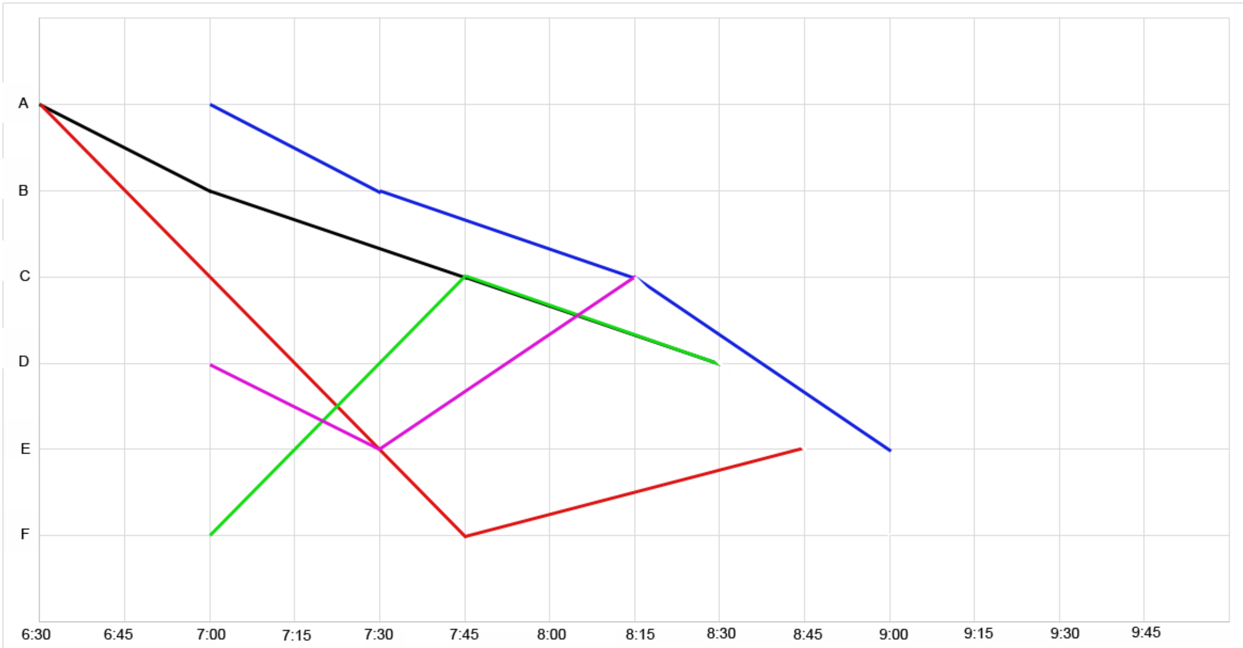

Figure 3: Time-space network schedule representation

routes are represented by a unique value of i. ETAs will vary for each possible route.

Index s represents the ETD of the flight. This is required if delaying flights or adjusting departure times forward in time, is to be considered. Each unique value of s represents a different departure time (ETD), and these will include as many different options as the airline will allow. Values of s will include multiples of the time discretisation chosen, in this case 15 minutes. Commodities are scheduled flights. Nodes relate to sectors.

Note that the simplified formulation shown here is very similar to that of Bertsimas and Patterson [?], although Bertsimas and Patterson did not specifically describe the use of a time-space network. Therefore, the formulation shown here relates to a time-space network and associated nomenclature.

Variables:

x_{fis} flight variable, binary, for flight f.

Parameters:

s take-off time period index, contained in set S
i route index, contained in set I
t time period, contained in set T

The model formulation is as follows:

Minimise $Z = \sum_{\text{all } F} \ c_x \, x_{fis}$

where c_x is the time the flight takes.

Constraints:

Arc (sector) capacities: $\sum_{t \in T} \chi^t_{fis} \leq A_q \, \forall \ q \ \in \ Q$,

where q is the sector number and Q the set thereof.

All scheduled flights flown: $\sum_{i \in I, \, s \in S} \ \chi_{fis} \ = \ 1 \forall f \in F$.

Other constraints required in certain instances would be to ensure continuation flights follow on from initial flights, and that a series of flights assigned to a certain aircraft are consecutive, with required turnaround time (ground time) between flights. These constraints would typically be included by adding ground arc variables and using node conservation of flow constraints. Since the formulation is NP-hard, if exact methods are used the problem size can become large and result in impractically long computational times.

5 ATFM rerouting problem

Disruptions can either be route or node related. A route disruption could be something like a weather event such as a storm, whereas a node disruption could be a delay in departure time, maybe due to an unexpected problem at the airport. If a disruption occurs, the current schedule might need to be rearranged accordingly, by assigning ground holding and airborne delays and/or rerouting flights.

A disruption has a starting time, which is likely to be in the near future or near past and a forecast ending time. For an arc disruption, there may be a defined geographical area that the disruption affects. For example, poor weather conditions will affect a certain geographical area, and might affect the capacity of a sector and/or an airport for a defined period of time.

In practice, delay and re-routing decisions are handled by air traffic managers. The goal of this work is to develop a technique to help with this process, by re-optimising a schedule after a disruption. The problem therefore will involve the time-space network from the time of the disruption to some affected time in the future (possibly the end of the schedule), and certain geographical sectors at certain times.

For the example in Figure 3, an arc disruption is introduced as shown in Figure 4 and shown as a shaded oval between sectors B and C, and between 7:30 and 8:00, for flight F4 (in blue). Such a disruption could cause the plane to take longer than expected on the planned route, or the plane could be rerouted. We assumed the

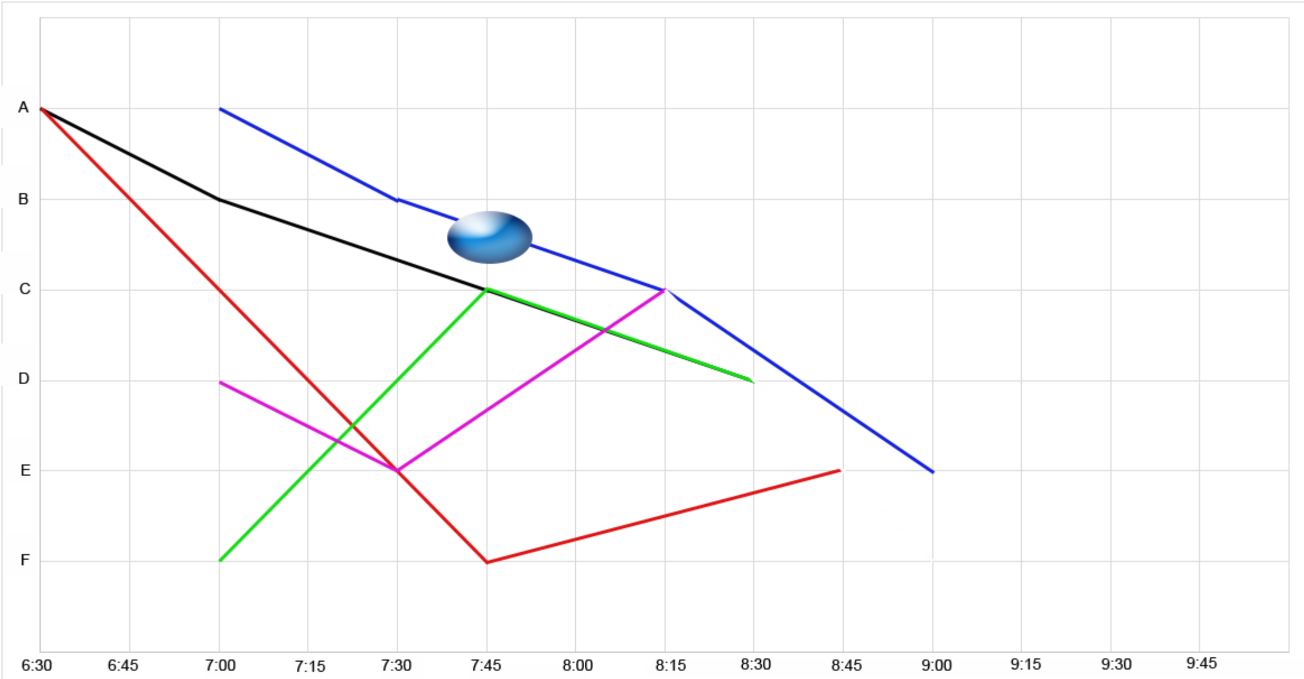

Figure 4: Arc disruption on time-space network

plane would be rerouted. We also assumed the disruption occurred at time 7:30, such that changes can only be made to the schedule after that time.

The rerouting must take into consideration the same constraints as the original management problem, that is, sector and airports capacities. A subsequent change in arrival time for F4 could also affect continuation flights, if any, and subsequent scheduled flights for the aircraft assigned to F4.

Changes can only be made to flights leaving after 7:30. Practically it would be beneficial to also not consider rerouting or delaying flights already being undertaken at that time. This leaves the time-space network after 7:30 to be considered. There are a limited number of flights that will be affected by a rerouting of F4, that is, only flights that share a flight path or origin or destination with either F4, or another flight which is being considered for delay or rerouting due to the rerouting of F4. In the spatial domain, there are a number of geographically distant flights that will not be affected and can be ignored.

6 Previous work

Agustin et al. [?] provide a comprehensive review of articles related to the ATFMP and related problems as of 2009. More current articles include Bertsimas and Patterson [?], Bertsimas and Patterson [?] and Bertsimas et al [?].

The ATFMP is dealt with by Bertsimas and Patterson [?] and Bertsimas et al

[?]. Bertsimas and Patterson [?] use integer programming techniques to solve the problem for all capacity limitations (arrival, departure and sector). The largest dataset was a real world instance of 1002 flights, 18 airports and 305 sectors over a time span of 8 hours and 5 min intervals, and solved in just over 8 hours of CPU time.

Bertsimas et al. [?] use integer programming to solve the ATFMP, but the method allows for rerouting decisions by adding constraints to the formulation. Instances of 6475 flights, 30 airports, 145 sectors and 22 time periods solved to within 1% integrality gap in 1 hour. Rerouting is included with additional constraints, and no additional variables, in the formulation.

Bertsimas and Patterson [?] deal with the ATFMRP specifically. They model a disruptive weather pattern such as a storm as it moves through a region, affecting a number of adjacent sectors. Multiple methodologies are used, essentially searching for a number of alternative aircraft routes, and choosing the optimal combination. Methods used include multi-commodity network flow integer programming, randomised rounding and a packing formulation. Multiple airlines with connected flight legs can be solved for, such that delays are evenly and fairly distributed. Ground delays and en-route delays (air holding) were used as control options, and not aircraft rerouting. The possibility of flight delay is rarely considered [?].

7 Methodology

The ATFMRP is discrete and can naturally be optimised using a neighbourhood search. For a disruption, the entire network does not have to be modelled, and only the flights after the current time and those geographically close need to be included. The objective is to reroute the aircraft at minimal additional cost. The original schedule would have been optimised before use. MATLAB code was written to both optimise the full schedule, and re-optimise the disrupted schedule.

7.1 Optimisation of full schedule

The methodology developed in this work is to create a starting, a current solution (possibly derived manually) and do a neighbourhood search for alternative, improved solutions. The current solution is then modified accordingly.

Infeasible arc sector capacity constraints were allowed in solutions to ensure a feasible solution was always possible. Also, it is highly likely that in reality, these could be treated as soft constraints, that is, that a few more flights in a sector than allowed by the constraint could in practise be handled by the sector controllers. For this work, these constraints were dealt with using the penalty method, that is, including infeasible constraints as penalties in the objective function of size proportional to the amount by which the constraint was exceeded.

Airport constraints were not specifically included, since they could be included as sector capacity constraints, and that is the assumption here. Routes or arcs which cannot be flown are dealt with by including them with high leg costs (times), such that they will not be used in a solution.

Three neighbourhood moves are used:

- Reroute The lowest cost reroute is chosen. A branch and bound search function considers all possible alternative routes.
- Delay n time periods A take-off delay of a certain number of time periods n is considered. In this work, a time period was set at 15 minutes, and delay periods of $n = 1, 2, 3$ and 4 were considered.
- Move flight take-off time (ETD) forward n time periods - Take-off time is moved forward in time a certain number of periods n. Periods of $n = 1, 2, 3$ and 4 were considered.

7.2 Re-optimisation after a disruption

To begin the process of re-optimisation , the cost of the affected arc is increased such that it becomes uneconomical in a solution, then the cost of alternative, local solutions are compared. The same neighbourhood search is used as for the full schedule optimisation, except now moves are only applied to a reduced set of flights. These flights are chosen according to:

- Whether they take off after the current time, current time being the time period ending before the time period during which the disruption occurs.
- Whether they share any flight paths with the affected flight.
- Whether they share any flight paths with the flights that may be affected by the affected flight.

Flights currently being carried out at the current time are not considered for modification. Therefore in-flight delays and rerouting of in-the-air flights are not considered.

7.3 Data

The manually created schedule in Table 1 was used. Maximum flow constraints on arcs (sector capacities) were set as a maximum of 3 flights in any sector during any time period.

The sector to sector traversing times are shown in Table 2. Penalty times are shown as 1005 minutes.

When an aircraft moves from one sector to another, the number of periods it will spend in each sector must be known, such that the number of aircraft in each sector in each time period can be calculated. These times are shown in Table 3.

Table 2: Sector to sector flight leg times in minutes

Sector	A	B	C	D	E	F
A	0	30	1005	1005	1005	75
B	30	0	45	1005	1005	45
C	1005	45	0	45	45	45
D	1005	1005	45	0	30	1005
E	1005	1005	45	30	0	60
F	75	45	45	1005	60	0

Table 3: Time in each sector for a sector to sector flight leg

From Sector	To Sector	Total Flight Time (min)	Flight Time in Sector 1 (min)	Flight Time in Sector 2 (min)
1	2	30	15	15
2	3	45	15	30
3	4	45	30	15
1	6	75	30	45
2	6	45	15	30
3	6	45	30	15
6	5	60	45	15
4	5	30	15	15
3	5	45	30	15
2	5	1005	495	510
2	4	1005	495	510
1	3	1005	495	510
1	4	1005	495	510
1	5	1005	495	510
4	6	1005	495	510

8 Results

The cost (time) of the manual schedule, before optimisation, is 5260 minutes. Capacity constraint penalties amounted to 34 (34 more flights per sector than allowed by the capacity constraints). After optimisation, the schedule cost was 2915 minutes, with capacity constraint penalties of 11. The optimised schedule is shown in Table 4.

Table 4: Optimised schedule

Flight	ETD	Origin	ETA	Destination	Sectors
1	06:30	A	08:35	D	ABCD
2	06:30	A	08:40	E	AFE
3	07:00	F	08:35	D	FCD
4	08:30	A	10:30	E	ABCE
5	08:00	D	10:00	F	DECF
6	06:45	D	08:15	E	DCE
7	07:15	D	10:00	A	DEFA
8	07:30	F	09:05	D	FCD
9	06:45	D	08:45	A	DCBA
10	08:30	D	10:00	F	DCF
11	07:30	A	09:35	D	ABCD
12	06:45	F	09:10	E	FBCE
13	07:15	A	09:25	E	AFE
14	08:00	D	09:25	F	DEF
15	07:15	F	09:15	A	FA
16	07:15	E	09:20	A	ECBA

Table 5: Re-optimised schedule after disruption

Flight	ETD	Origin	ETA	Destination	Sectors
1	06:30	A	08:35	D	ABCD
2	06:30	A	08:40	E	AFE
3	06:45	F	08:30	D	FCD
4	09:00	A	11:00	E	ABCE
5	08:00	D	10:00	F	DECF
6	06:45	D	08:15	E	DCE
7	07:15	D	10:00	A	DEFA
8	08:30	F	10:00	D	FCD
9	06:45	D	08:45	A	DCBA
10	08:30	D	10:00	F	DCF
11	07:30	A	09:35	D	ABCD
12	06:45	F	09:10	E	FBCE
13	07:15	A	09:25	E	AFE
14	08:00	D	09:25	F	DEF
15	07:15	F	09:15	A	FA
16	07:15	E	09:20	A	ECBA

After the disruption, the re-optimised portion of the schedule had a cost of 2515 minutes, with 7 penalties. The schedule is shown in Table 5.

9 Discussion

A shortcoming of this work was the absence of the availability of real datasets. Real datasets would surely be larger than the artificial dataset used here, and would provide better guidance as to what further work is required for this model. The use of real datasets would also have individual aircraft assignments to series of flights, and use of node conservation of flow constraints would become necessary.

The developed neighbourhood search method proved useful to improve the full schedule and to re-optimise the disrupted schedule. One issue which could be addressed in future research is the fact that if a flight is delayed or the take-off time moved forward, the model does not assign a cost to that delay or take-off time adjustment. In practise, such changes would result in some cost to an airline, possibly an indirect cost due to reduced customer satisfaction. For example, flight 4 was delayed 1.5 hours (7:00 to 8:30) from the initial schedule to the optimised schedule, then a further 30 minutes after the disruption, in the re-optimised schedule. A cost of delay could easily be added to the objective function to minimise these effects.

The simple neighbourhood moves used in this work are effective in reducing the schedule cost. A future improvement would be the addition of a Tabu list, such that the true optimum for a schedule is more likely to be found. This may be useful for larger, real-life problems.

More neighbourhood moves could be developed, depending on the rate of convergence for larger instances. For example, combined rerouting of two or more flights, combined delays or a combination of moves could be considered. In-flight delays could also be considered.

Although the methods introduced here are shown to work in that they reduce the schedule costs and reduce infeasibility, results from an exact method need to be obtained for comparison and validation.

10 Conclusions

Novel methods of solving the ATFMP and the ATFMRP have been presented. Tests on a small instance have shown that the methods work and are practical for use in industry.

References

[1] D. Bertsimas, S. S. Patterson: The air traffic flow management problem with enroute capacities. Operations Research **46**, 1998, 406-422.

[2] A. Agustin, A. Alonso-Ayuso, L. F. Escudero and C. Pizzaro: Mathematical optimization models for air traffic flow management: A Review. 2009.

[3] D. Bertsimas, and S. S. Patterson: The traffic flow management rerouting problem in air traffic control: A Dynamic Network Flow Approach. Transportation Science **34**, 2000, 239-255.

[4] D. Bertsimas, G. Lulli and A. R. Odoni: An integer optimization approach to large-scale air traffic flow management. Operations Research **59**, 2011, 211-227.

[5] P. Belobaba, A. Odoni, C Barnhart: The global airline industry. John Wiley & Sons, UK, 2009.

[6] http://www.slideshare.net/aulger/air-traffic-control-organization-lesson-presentation ATC Lesson, Dept. of Aviation, Ohio University. Accessed 20 Aug. 2016.