PROCEEDINGS OF THE HYBRID MATHEMATICS IN INDUSTRY STUDY GROUP

2022

Hybrid Mathematics in Industry Study Group
South Africa

MISGSA 2022

The writing of a Technical Report for the Proceedings of the MISGSA was coordinated by the moderator of the problem. Sections of the Report were written by the moderator and by other members of the study group who worked on the problem.

The Editor of the Proceedings was

Prof D P Mason (University of the Witwatersrand, Johannesburg)

The Technical Reports were submitted to the Editor. Each Report was refereed by one referee. On the recommendation of the referees the Reports were accepted for the Proceedings subject to corrections and minor revisions. The Editor would like to thank the referees for their assistance by refereeing the Reports for the Proceedings.

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PREFACE

The nineteenth Mathematics in Industry Study Group (MISG) in South Africa was organised by the University of the Witwatersrand, Johannesburg, from Monday 7 February to Friday 11 February 2022. It was a Hybrid MISG with some staff and students participating online and some onsite. There were onsite Study Group meetings at the University of the Witwatersrand and at the University of Zululand.

The total number of registered participants at the MISG was sixty-eight. There were twenty-seven Academic Staff, thirty-four Graduate Students, six Industry Representatives and one Technical Administrator. The invited guests were:

Neville Fowkes	University of Western Australia, Australia
Graeme Hocking	Murdoch University, Perth, Australia
Timothy Myers	Centre de Recerca Matematica, Barcelona, Spain
Alba Cabrera Codony	University of Girona, Spain
Fatima Nouri	Badji Mokhtar University, Algeria
Denis Ndanguza	University of Rwanda, Rwanda
Hilary Ockendon	University of Oxford, United Kingdom
John Ockendon	University of Oxford, United Kingdom

The South African Universities and Institutes which were represented were:

African Institute for Mathematical Sciences
Durban University of Technology
North-West University
Rhodes University
Sol Plaatjie Univeristy
University of Johannesburg
University of Mpumalanga
University of Pretoria
University of the Witwatersrand
University of South Africa
University of Zululand

A Hybrid MISG allows participation of researchers who otherwise would find it difficult to participate in the MISG. This includes experts in the field who are not needed for the whole of the meeting, those with travel difficulties within South Africa and internationally, those with caring responsibilities and teaching commitments.

For both the Graduate Modelling Camp and the Study Group the software Zoom was used for all virtual meetings, both the main group and the breakaway study groups. An interactive Whiteboard was used for discussing mathematics. SharePoint online was used for depositing and sharing papers and other important documents. A copy of the MISG 2022 Technical Information Guide, containing the Link to Zoom and all necessary information related to Zoom and SharePoint was sent to all participants before the meeting.

All participants had access to the necessary technology, either at home or at a University.

The Hybrid MISG was officially opened on Monday morning by Professor Nithaya Chetty, Dean of Science, University of the Witwatersrand.

South African industry had been approached to submit problems during 2021. Five problems were submitted. On Monday morning each Industry Representative made a twenty-five minute virtual presentation in which the problem was described and outlined. The academics and graduate students then split into small study groups and worked in either onsite venues or virtual break away rooms on the problem of their choice. Each problem was co-ordinated by an academic moderator and one or more student moderators. The role of the academic moderator was to co-ordinate the research on the problem during the week of the meeting and also to do preparatory work including literature searches before the meeting. The main function of the student moderators was to present short virtual presentations at the end of each working day reporting on the progress made that day. The moderators were in contact with the Industry

Representatives throughout the meeting. On Friday morning there was a full virtual report back session to industry. Each senior moderator, with assistance from the student moderators, made a twenty-five minute virtual presentation, summing up the progress made and the results that were obtained. Each Industry Representative then had five minutes to comment on the progress and the results that were reported. The MISG ended at lunch time on Friday.

The MISG was preceded by a Hybrid Graduate Modelling Camp from Wednesday 2 February to Saturday 5 February 2022. The objective of the Graduate Modelling Camp was to enable the graduate students to familiarize themselves with the Zoom technology and to provide them with the necessary background to make a positive contribution to the MISG the following week. The students were given hands-on experience at working collaboratively and interacting scientifically in small onsite or virtual study groups on problems of industrial origin and at making virtual presentations on their findings. Five problems were presented to the graduate students. The problems and presenters were:

• Designing a distributed personal-medical system	Professor Jeff Sanders African Institute for Mathematical Sciences and University of Stellenbosch
• An investigation of some numerical methods for solving singular integral equations with Cauchy type kernels	Dr Mathibele Nchabeleng University of Pretoria
• Solitary waves in mixtures of fluid and gas bubbles	Professor David Mason and Dr Thama Duba University of the Witwatersrand
• Mathematical modelling of the Max 2-Cut Problem and solving the relaxed model	Professor Montaz Ali University of the Witwatersrand
• Detecting oil and gas using sound waves	Mr Erich Mubai University of the Witwatersrand

The graduate students worked in small onsite or virtual study groups on the problem of their choice.

The sponsors of the Hybrid Graduate Modelling Camp and the Hybrid MISG were:

- DST-NRF Centre of Excellence in Mathematical and Statistical Sciences
- African Institute for Mathematical Sciences
- Hermann Ohlthaver Trust

We thank the sponsors without whose support the Graduate Modelling Camp and the MISG could not have taken place.

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PROBLEM STATEMENTS

Problem 1. **Diffuser optimum imbibition**

Industry: Sugar Cane Processing

Industry Representative: Richard Loubser, Sugar Milling Research Institute, University of KwaZulu-Natal, Durban

Problem statement:

Typical sugar cane consists of 70% water, 15% insoluble material referred to as fibre and 15% dissolved solids. What is reported as fibre is mostly made up of fibrous material from the cane stalk and leaves, but also includes small amounts of soil and other insoluble components. The dissolved solids are mostly made up of sucrose with smaller amounts of other components such as monosaccharides and inorganic salts. The dissolved solids are extracted from the plant and concentrated by boiling off water to produce a syrup. Crystal sugar is produced during additional concentration steps in a process called pan boiling. Before sugar-bearing juice can be extracted from the cane, cane stalks are prepared by shredding with a hammer-mill shredder. This reduces the cane to short fibres where the sap or juice within the cells is exposed. This juice contains the sucrose. The extraction process involves washing the sugar-containing juice out of the shredded cane. The most common type of extraction unit in South Africa is a diffuser, which is a staged counter current leaching process. Additional water, called imbibition, is added to facilitate washing. The extraction process has two mechanisms: free juice is washed off the prepared cane in each stage of the diffuser by the liquid that passes over the fibres, and diffusion of sucrose through the walls of unbroken cells within the cane fibre. The free juice is much easier to separate from the fibre and dominates the extraction process. These processes are driven by the difference in concentration of juice attached to the fibre and the juice percolating through the bed as well as the contact between the flowing juice and the fibre. The capacity of the extraction plant is considered to be limited by fibre throughout. Consequently, the amount of imbibition added is calculated and reported as a percentage of the fibre. Any water added to the process needs to be evaporated before the sugar can be crystallised. This evaporation requires energy that comes from burning fuel in a boiler. Much of the energy is supplied by burning the spent fibre after sucrose has been extracted, known as bagasse. If there is insufficient bagasse, the fuel has to be supplemented with coal. Burning of coal adds to the production cost so the trend has been to reduce the amount of imbibition used so as to control energy costs. Common wisdom suggests that reduction in the amount of added imbibition water results in reduced sucrose recovery efficiency, reported as extraction. The reduction of imbibition has been associated with reduced extraction. The question is whether this is the consequence of a fundamental dependence of extraction on imbibition rate, or whether changes to diffuser operational parameters or configuration could allow high extraction efficiency at reduced imbibition rates.

The diffuser is divided into around 14 stages with prepared cane moving from stage 1 to stage 14. Imbibition is applied by pouring water over the cane at the start of the last stage. Counter current flow is achieved by collecting juice in a tray below the prepared cane bed in each stage and pumping it to a spray above the bed in advance of where it was collected. The spray is positioned so that the majority of the juice will percolate through the bed and fall into the previous stage. Some of the juice, however, returns to the same tray. This is known as recycle. The effect of the recycle is to increase the amount of water held up in the stage, i.e. the mass of juice resident in that stage at a particular point in time. Greater juice hold-up results in increased juice percolation rates (volume of juice per area of cane bed), a property that has been experimentally shown to influence extraction efficiency. The recycle of juice, however, may increase the amount of dissolved sugar in the juice within a state, relative to a condition with less recycle. This may compromise the efficiency of sucrose extraction from the cane. Achieving optimum extraction therefore depends on a balance between factors such as quantity of juice hold-up and sucrose concentrations in that juice.

We are looking for an indication whether reducing imbibition will inevitably result in a penalty in extraction efficiency, or whether adjustments can be made to parameters, such as spray position (or the bed speed/height relationship), that will compensate for the reduction in imbibition

Problem 2. Tourism sector recovery plan for airlines

Industry: Tourism Sector

Industry Representative: Dr Lombuso Precious Shabalala, University of Mpumalanga

Problem statement:

The 1996 White Paper on the Development and Promotion in South Africa [1] provides for the promotion of domestic and international tourism. The National Development Plan identifies tourism as a labour-intensive sector with the potential to stimulate economic growth and transformation.

According to Saayman [2] Tourism is about movement of people from one place to another. The need or desire to travel takes place for various reasons which includes business, leisure and visiting friends and relatives. Transport is used to effect the movement. The movement takes place within the eight industries that form the tourism sector. The movement takes place within the eight industries that form the tourism sector. The transport industry makes a vital contribution to the total tourism experience.

The problem focuses on air transport.

As part of the Tourism Sector Recovery Plan COVID-19 Response [3] aiming to ignite the tourism sector, optimisation of profit through reduction in tax on available seats was suggested. Questions that need to be answered are:

1. What changes need to be implemented?
2. How many seats need to be sold and the cost per seat taking note that due to COVID-19 regulations airlines are not permitted to carry a full capacity.
3. How can airlines find ways to increase their profit using the available seats under the given COVID-19 regulations and taking into account the new Omicron variant?

References

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2. Saayman, M(2013). En Route with Tourism – An introductory, Juta.

3. National Department of Tourism (2021). Tourism Sector Plan COVID-19 Response. Available at:

https://www.tourism.gov.za //AboutNDT/Documents%20Sector%Recovery%20

Problem 3. Scaling up from experiment to industry

Industry: Carbon Capture

Industry Representatives: Dr Alba Cabrera, Laboratori d'Enginyeria Quimica I Ambiental, Giron, Spain
Professor Tim Myers, Centre de Recerca Matematica, Barcelona, Spain

Problem Statement

When developing new processes which are to be employed on a large scale it is much more cost effective to test on small scale devices and subsequently scale. However, simply scaling dimensions seldom leads to the same behavior predicted by the test device. Consequently the process of scale-up is non-trivial and often contentious.

The problem will focus on a specific configuration,where a fluid is passed through a cylindrical column containing a porous adsorbent material. Column adsorption is employed in a variety of areas including carbon capture and the removal of many environmental contaminants.

A typical set of industrial adsorption columns has dimensions of the order of meters while in a typical test the tube diameter is around 1.5 cm.

Industrial columns typically contain pellets of adsorbent material. Pellets of activated carbon typically have diameter of 4 mm. For experiments these are ground into a powder with micron dimensions.

The mathematical model for column adsorption is relatively straightforward, involving an advection-diffusion equation coupled to a linear mass sink which represents the adsorption. The question for the study group will be to define parameters, such as flow regime, particle size, column dimensions, and examine wall effects, which will allow results from small scale tests to guide and understand the large scale process. Alternatively, how can the tests be related to the industrial set-up?

Problem 4. Movement of bubbles in a tube for methane extraction in Lake Kivu

Industry: Generation of Electricity

Industry Representative: Professor Denis Ndanguza, College of Science and Technology, University of Rwanda, Kigali, Rwanda

Problem Statement:

Lake Kivu is a fresh water lake and along with Cameroonian Lake Nyos and Lake Monoun, is one of three that are known to undergo limnic eruptions. Around the lake, geologists have found evidence of massive local extinctions about every thousand years, presumably caused by out gassing events. The trigger for lake overturns in Lake Kivu is unknown, but volcanic activity is suspected. The gaseous chemical composition of exploding lakes is unique to each lake. In Lake Kivu's case, it includes methane and carbon dioxide, as a result of lake water interaction with volcanic hot springs.

The amount of methane contained at the bottom of the lake is estimated to be 65 cubic kilometers. If burnt over one year, it would give an average power of about 100 gigawatts for the whole period. The lake also holds an estimated 256 cubic kilometres of carbon dioxide which, if released in an eruption event, could suffocate all of the inhabitants of the lakeshore. The methane is reported to be produced by microbial reduction of the volcanic carbon dioxide. A future overturn and gas release from the deep waters of Lake Kivu would result in a catastrophe, dwarfing the historically documented lake overturns at the much smaller Lakes Nyos and Monoun. The lives of the approximately two million people who live in the lake basin area would be threatened.

To overcome this threat, the government of Rwanda is reducing the methane gas by converting it into electrical power. A tube is injected into the resource zone (Layer 4) and due to high pressure, bubbles are formed and their buoyance drives the upward flow in the tube. We are interested to study the bubble formation, bubble growth and movement within the tube. (Prior knowledge of its diameter is an added value). When the temperature is increased sufficiently such that the water becomes saturated with the dissolved gases, what determines where bubbles will first form? Will bubbles form near the glass, along the plastic, or randomly along the length? What determines the bubble size?

Bubbles that form in the tube will grow to a size inconsistent with the amount of dissolved gas initially present in the water in the tube. This is due to diffusion of air into the tube which adds to the volume of air that already is in the bubbles.

Problem 5: The drop shunting problem

Industry: Beverage

Industry Representative: Professor Stephen Visagie, University of Stellenbosch

Problem Statement:

The drop shunting problem will only happen within cities or at least big densely populated areas. The company has truck and trailer combinations but it has more trailers than trucks. A trailer is loaded at the depot with the bottles of beverage for a number of clients. A truck takes it to the first drop off point and leaves it there for that client's stock to be offloaded and the empty bottles to be loaded. While that happens the truck moves to another trailer that is finished at another client and moves the trailer to its next client of takes an empty trailer back to the depot or collects another full trailer from the depot and takes it to a client. One thus has a pool of trucks shunting a pool of trailers around between clients and the depot.

How do we do this while travelling the least distance?

Of course the trucks have a fixed cost of higher and therefore it is cheaper to keep them moving and shunting trailers around rather than just waiting at the depot or at each client while the trailers get loaded or unloaded. There are a whole number of questions here:

What is the right number of trucks and trailers?
How should the deliveries be structured to minimise total cost?

One can assume a predetermined demand at each client. This problem also has a time component to it because the trailers must have spent different minimum times at each point before the next truck can come and take it to its next destination. This minimum time would be roughly proportional to the size of the loading/unloading. And of course it takes time for the trucks to move as well. Trailers cannot stand overnight at a client and therefore they must come back to the deport by close of day.