Thin Film Conductivity Measurements

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Reminder of Experimental Setup

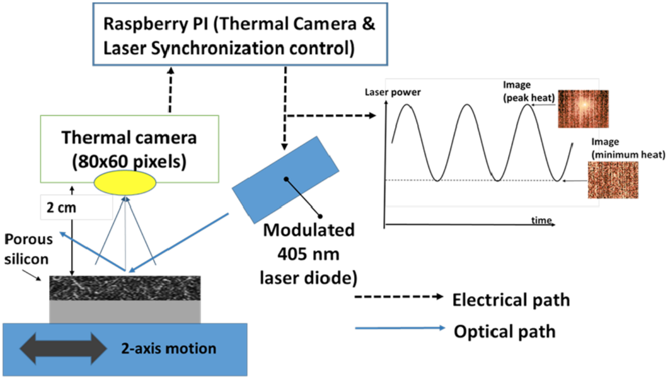

Figure 1: Optical system diagram for the thermography system developed as part of this work to investigate thermal characteristics of porous silicon. The physical housing of the system within a 20×20×13 cm³ IP66 rated enclosure.

Approach Taken

- Understand fundamentals of actual situation
- Attack a simplified problem
  - 1-D version
  - No Heat loss to substrate or air
- Model heat loss in 1-D version

Physical Setup

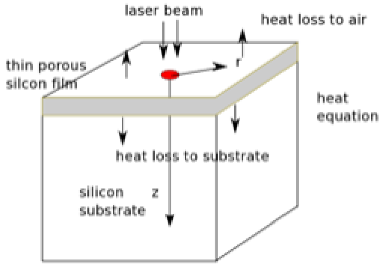

Figure 1: Simplified view of experiment

Fundamental equations

$$\frac{\partial T}{\partial t} = \kappa_f \nabla^2 T + \frac{Q_0}{h} \delta(r) e^{i\omega t} \qquad \text{Film}$$

$$\frac{\partial T}{\partial t} = \kappa_s \nabla^2 T \qquad \text{Substrate}$$

Complications

- Thin Layer
- Conductivity Ratio About $100 : 1$
- Hard To Asses Heat Loss To Atmosphere

Aim

How best to determine thermal properties of film
(what ω to use ....)

Wolf's arrangement

A simpler arrangement has been explored by Wolf.

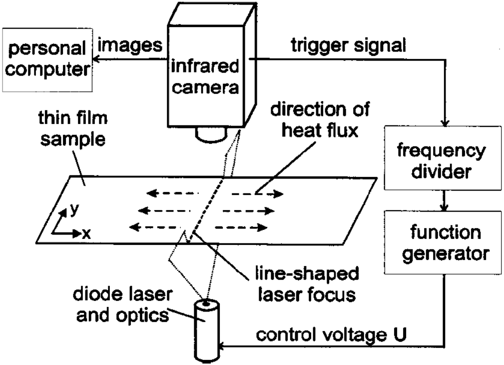

FIG. 1. Experimental setup: A sinusoidally modulated diode laser illuminates the backside of a thin film sample. The focus of the laser light is line shaped. The induced thermal wave is visualized by a lock-in procedure using an IR camera. The dashed arrows symbolize the direction of the heat flux.

Figure 2: Wolf cuts off the film and heats along a line: much harder experimentally, *much* easier analytically

Equations

$$\mathrm{Heat\ \ Equation:} \quad \rho_f C_f T_t(x,t) = k_f T_{xx}(x,t) \tag{1}$$

$$\mathrm{Heat\ \ Input:} \quad -k_f T_x(x,t)\Big|_{x=0} = q_0 e^{i\omega t} \tag{2}$$

We expect a steady state (or oscillatory) solution of the form

$$T(x,t) = e^{i\omega t} X(x) \tag{3}$$

Scaling

Set

$$\begin{aligned} T &= T_o + \Delta T\, T' \\ \omega t &= t' \\ x &= x_0 x' \end{aligned} \tag{4}$$

Substituting (4) into (1) we obtain

$$\rho_f C_f \Delta T \omega = \frac{k_f \Delta T}{x_0^2} T'_{x'x'} \tag{5}$$

This leads to

$$x_0^2 = \frac{K_f}{\omega} \Rightarrow x_0 = \sqrt{\frac{K_f}{\omega}} \tag{6}$$

with K_f the diffusivity given by

$$K_f = \frac{k_f}{\rho_f C_f} \tag{7}$$

The scaled result is

$$T'_{t'} = T'_{x'x'} \tag{8}$$

Scaling

Substituting (4) into (2) we obtain

$$-\frac{k_f \Delta T}{x_0} \frac{\partial T'}{\partial x'} = q_0 e^{it'} \qquad (9)$$

This leads to

$$\Delta T = \frac{q_0}{\sqrt{k_f \rho_f C_f \omega}}$$

The result of scaling is

$$\left.\frac{\partial T'}{\partial x'}\right|_{x=0} = -e^{it'}$$

Observations

$$x_0 \propto \frac{1}{\sqrt{\omega}}$$

$$x_0 \propto \sqrt{K_f}$$

$$\Delta T \propto \frac{1}{\sqrt{\omega}}$$

$$\Delta T \propto q_0$$

The substitution of (3) into (8) gives

$$X_{xx} - iX = 0 \tag{10}$$

with

$$X(x) = Ae^{-\frac{1}{\sqrt{2}}(1+i)x} \tag{11}$$

with Constant of integration A given by

$$A = \frac{\sqrt{2}}{(1+i)} \tag{12}$$

Thus,

$$X(x) = \frac{\sqrt{2}}{(1+i)} e^{-\frac{1}{\sqrt{2}}(1+i)x} \tag{13}$$

Hence,

$$T\left(x,t\right) = e^{\frac{-x}{\sqrt{2}}} e^{i\left(t - \frac{-x}{\sqrt{2}} - \frac{\pi}{4}\right)} \tag{14}$$

Results For No Heat Loss Case (*Insulated film*)

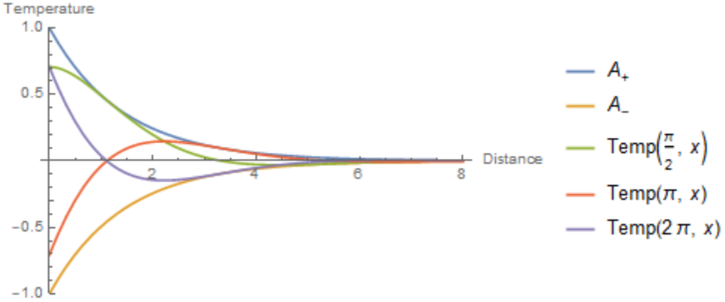

Figure 3: Temperature Profiles for different t

Heat Losses Case

$$\rho_f C_f T_t = k_f T_{xx} - \gamma(T - T_0) \quad (15)$$
$$-k_f T_x = q_0 e^{i\omega t} \quad (16)$$

with the same scaling we obtained

$$T'_{t'} = T'_{x'x'} - \mu T'$$
$$-T'_{x'} = e^{it}$$

where μ is scaled heat loss parameter and it is

$$\mu = \frac{\gamma}{\rho_f C_f \omega}$$

.

Thus,

$$X(x) = \frac{1}{\sqrt{i+\mu}} e^{(\sqrt{i+\mu})x} \quad (17)$$
$$T(x,t) = e^{it} \frac{1}{\sqrt{i+\mu}} e^{(\sqrt{i+\mu})x} \quad (18)$$

$$\sqrt{\mu + i} = \sqrt{r_0}\left[\cos\left(\frac{\pi}{4} - \frac{\phi}{2}\right) + i\sin\left(\frac{\pi}{4} - \frac{\phi}{2}\right)\right] \tag{19}$$

where $\tan(\phi) = \mu$.
Hence,

$$X(x) = \frac{1}{\sqrt{\mu + i}}\left[e^{-\sqrt{r_0}\left[\cos\left(\frac{\pi}{4} - \frac{\phi}{2}\right)x\right]} e^{-i\sqrt{r_0}\left[\sin\left(\frac{\pi}{4} - \frac{\phi}{2}\right)x\right]}\right] \tag{20}$$

As $\mu \to 0$, we go back to the simple case.
(20) approximately gives

$$AMP(x) = \left[1 - \left(\frac{\phi^2}{2}\right)\right] e^{\frac{-1}{\sqrt{2}}\left(1 + \frac{\phi}{2}\right)x} \tag{21}$$

Results For Heat Loss Case (*Not insulated*)

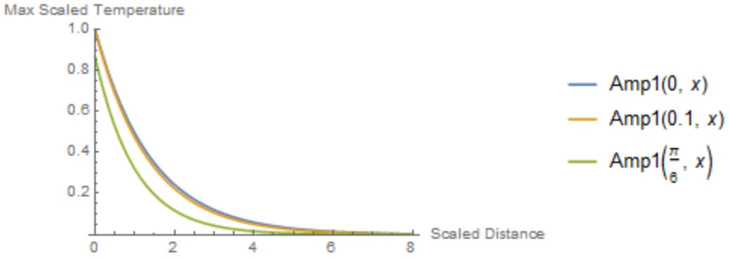

Figure 4: Temperature Profiles for different t

Conclusion

We presented the general problem and examined the situation in which heat was supplied along a line:

- In the special ideal case in which there was no heat loss, we found that waves travels away from the line source with decaying amplitude
- The maximum temperature change reached was

$$\frac{q_0}{\sqrt{k_f \rho_f C_f \omega}}$$

- These results can be used to determine K_f, k_f and $\rho_f C_f$
- We modelled effect of heat losses and found formulae that would be used to account for the heat losses
- If heat loss parameter is less than 0.3, the effect is minor.