MISG 2011, Problem 1:
Coal Mine pillar extraction

Group 1 and 2

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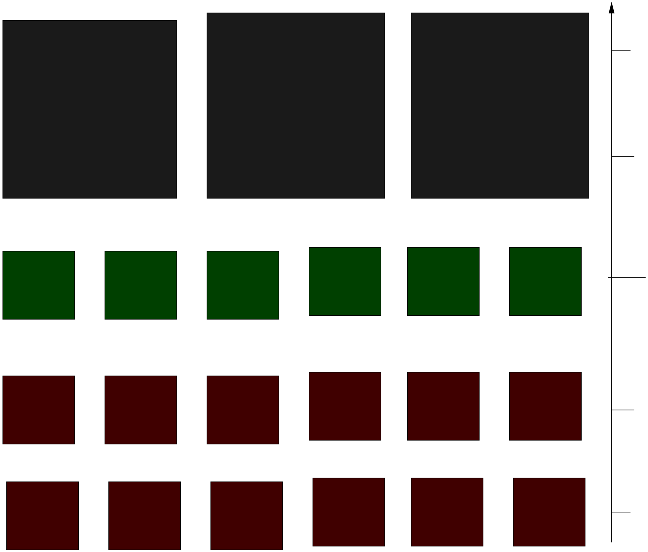
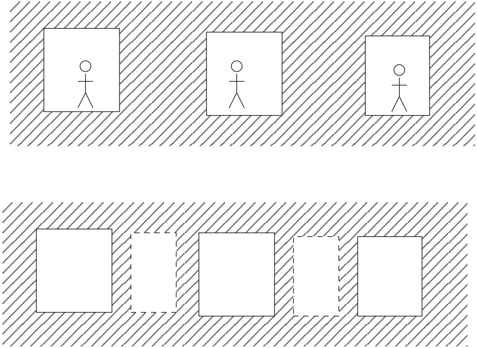
A small coal mining developpement



Introduction



- Cutting pillars to make snooks
- QUESTION: How do snooks collapse in order to have a safe and controlled collapse of the rock?

Two approaches

- How do snooks collapse? “ The pillar problem”
- How stable is the roof when snooks collapse ’strut-beam analysis’

The set up

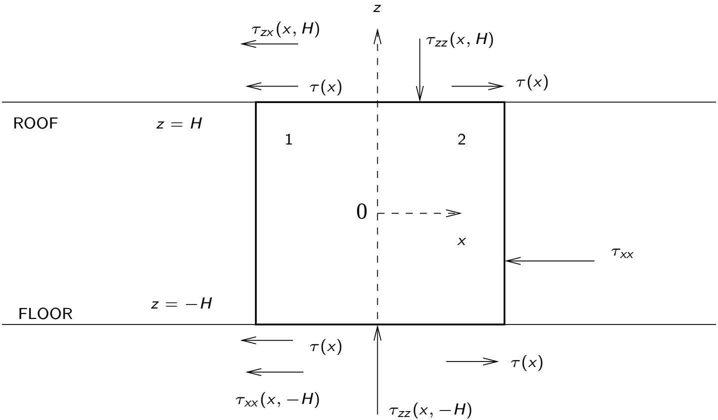


Figure: The stresses in a pillar

Governing equations

The stresses are considered in the xz-plane in the form

$$\tau_{xx} = \tau_{xx}(x,z), \quad \tau_{xz} = \tau_{xz}(x,z), \quad \tau_{zz} = \tau_{zz}(x,z)$$

and the average stress is defined as

$$\bar{\tau}_{ik}(x) = \frac{1}{2H}\int_{-H}^{H} \tau_{ik}(x,z)dz.$$

The equations of static equilibrium in 2D reads

$$\frac{\partial}{\partial x}\tau_{xx} + \frac{\partial}{\partial z}\tau_{zx} = 0 \quad (1)$$

$$\frac{\partial}{\partial x}\tau_{xz} + \frac{\partial}{\partial z}\tau_{zz} = 0 \quad (2)$$

Taking the averages in the previous equations, we arrive at

$$\frac{d}{dx}\bar{\tau}_{xx} + \frac{1}{2H}\left[\tau_{zx}(x,H) - \tau_{zx}(x,-H)\right] = 0 \quad (3)$$

$$\frac{d}{dx}\bar{\tau}_{xz} + \frac{1}{2H}\left[\tau_{zz}(x,H) - \tau_{zz}(x,-H)\right] = 0 \quad (4)$$

Governing equations and BCs for Region 1

- For $-1 \leq \bar{x} \leq 0$,

$$\frac{d^2}{d\bar{x}^2}\bar{\tau}_{xx}^{(1)} + \frac{3}{\mu}\frac{L}{H}\frac{d}{d\bar{x}}\bar{\tau}_{xx}^{(1)} + 3m\left(\frac{L}{H}\right)^2 \bar{\tau}_{xx}^{(1)} = -3\left(\frac{L}{H}\right)^2 \tag{5}$$

with

$$m = \left[(1+\mu^2)^{\frac{1}{2}} + \mu\right]^2 > 1 \tag{6}$$

and μ the coefficient of internal friction, $\bar{x} = \frac{x}{L}$.

$$\bar{\tau}_{zz} = c_0 + \mu\bar{\tau}_{xx}.$$

and the boundary condition

$$\bar{\tau}_{xx}^{(1)}(-1) = 0 \quad \frac{d\bar{\tau}_{xx}^{(1)}}{d\bar{x}}(0) = 0. \tag{7}$$

Governing equations and BCs for Region 2

- For $0 \leq \bar{x} \leq 1$,

$$\frac{d^2}{d\bar{x}^2}\bar{\tau}_{xx}^{(2)} - \frac{3}{\mu}\frac{L}{H}\frac{d}{d\bar{x}}\bar{\tau}_{xx}^{(2)} + 3m\left(\frac{L}{H}\right)^2\bar{\tau}_{xx}^{(2)} = -3\left(\frac{L}{H}\right)^2 \tag{8}$$

with the boundary condition

$$\bar{\tau}_{xx}^{(2)}(1) = 0. \tag{9}$$

Moreover, the following compatibility conditions hold

$$\bar{\tau}_{xx}^{(1)}(0) = \bar{\tau}_{xx}^{(2)}(0) \tag{10}$$

$$\frac{d\bar{\tau}_{xx}^{(2)}}{d\bar{x}}(0) = 0. \tag{11}$$

Solution of the Mohr-Coulomb constitutive equations

From the characteristic equations of the ODEs, we get the critical value of μ,

$$\mu_{\mathrm{crit}} \doteq \sqrt{\frac{3}{4(1+\sqrt{3})}} \approx 0.52394565828751.$$

- $\mu < \mu_{\mathrm{crit}}$,

$$\bar{\tau}_{xx}^{(1)} = \frac{1}{m}\frac{\lambda_2 \exp(-\lambda_1 \bar{x}) - \lambda_1 \exp(-\lambda_2 \bar{x})}{\lambda_2 \exp(\lambda_1) - \lambda_1 \exp(\lambda_2)} - \frac{1}{m}, \tag{12}$$

and

$$\bar{\tau}_{xx}^{(2)} = \frac{1}{m}\frac{\lambda_1 \exp(\lambda_2 \bar{x}) - \lambda_2 \exp(\lambda_1 \bar{x})}{\lambda_1 \exp(\lambda_2) - \lambda_2 \exp(\lambda_1)} - \frac{1}{m}, \tag{13}$$

Solution of the Mohr-Coulomb constitutive equations

- If $\mu = \mu_{\text{crit}}$,

$$\bar{\tau}_{xx}^{(1)} = \frac{1+\alpha\bar{x}}{m(1-\alpha)}\,\mathrm{e}^{-\alpha(1+\bar{x})} - \frac{1}{m}, \tag{14}$$

and

$$\bar{\tau}_{xx}^{(2)} = \frac{1-\alpha\bar{x}}{m(1-\alpha)}\,\mathrm{e}^{-\alpha(1-\bar{x})} - \frac{1}{m}, \tag{15}$$

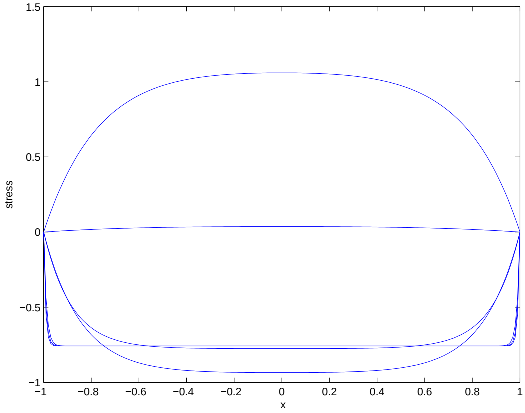
- If $\mu > \mu_{\text{crit}}$,

$$\bar{\tau}_{xx}^{(1)} = \frac{\tilde{\beta}\cos\tilde{\beta}\bar{x} + \alpha\sin\tilde{\beta}\bar{x}}{m(\tilde{\beta}\cos\tilde{\beta} - \alpha\sin\tilde{\beta})}\,\mathrm{e}^{-\alpha(1+\bar{x})} - \frac{1}{m}, \tag{16}$$

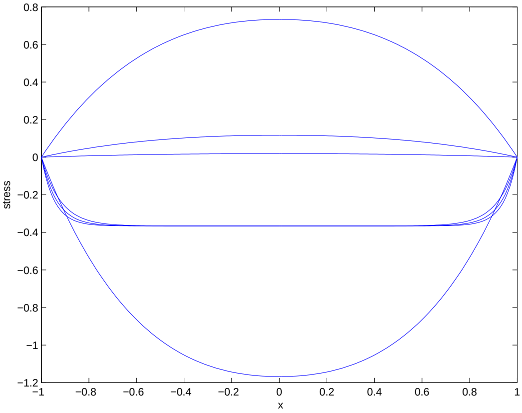
and

$$\bar{\tau}_{xx}^{(2)} = \frac{\tilde{\beta}\cos\tilde{\beta}\bar{x} - \alpha\sin\tilde{\beta}\bar{x}}{m(\tilde{\beta}\cos\tilde{\beta} - \alpha\sin\tilde{\beta})}\,\mathrm{e}^{-\alpha(1-\bar{x})} - \frac{1}{m}. \tag{17}$$

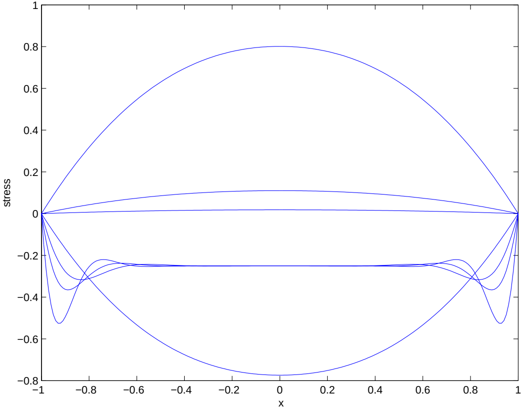
Analytical solutions
Case 1



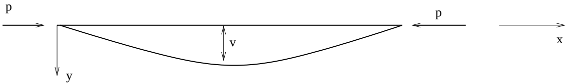
Analytical solutions
Case 2



Analytical solutions
Case 3



Buckling of a Strut



- Two equations for the *bending moment* at point x along the *strut*

$$EI\frac{d^2v}{dx^2} = M(x), \quad M(x) = -pv(x). \tag{18}$$

- These equations gives

$$\frac{d^2v}{dx^2} + \frac{p}{EI}v(x) = 0. \tag{19}$$

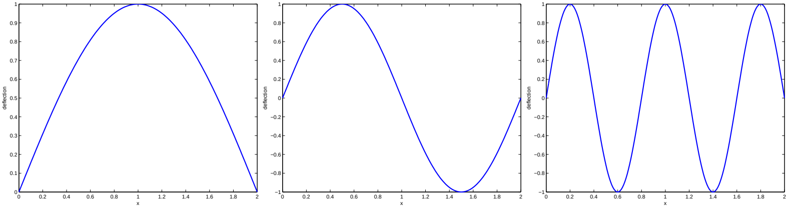
- Solving (19) with the corresponding boundary conditions

$$v(0) = 0, \qquad v(L) = 0,$$

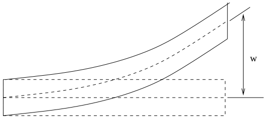
gives

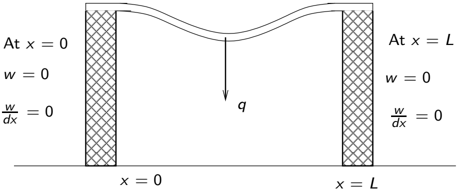
$$v(x) = A\sin(\frac{n\pi}{L}x), \quad n = 1, 2, 3, \cdots \tag{20}$$

Buckling of a Strut
Numerical results



Euler-Bernoulli Beam





$$\frac{d^2}{dx^2}(EI\frac{d^2w}{dx^2}) = q \tag{21}$$

Assumptions

- Mass is uniformly distributed.
- The beam is composed of an isotropic material.

$$-EI\frac{d^2w}{dx^2} = M \qquad \text{Bending moment} \tag{22}$$

$$-\frac{d}{dx}(EI\frac{d^2w}{dx^2}) = Q \qquad \text{Shear force on the beam} \tag{23}$$

Euler-Bernoulli Beam

- Now the deflection of the beam satisfies the equation

$$\frac{d^4w}{dx^4} = \frac{q}{EI} \tag{24}$$

- subject to the boundary conditions

$$w(0) = w(L) = 0 \qquad \frac{dw}{dx}(0) = \frac{dw}{dx}(L) = 0 \tag{25}$$

- The solution is found as

$$w(x) = \frac{q}{24EI}x^2(x-L)^2. \tag{26}$$

- The curvature is given by

$$C = |-\frac{q\,L^2 - 6\,q\,x\,L + 6\,q\,x^2}{12}|$$

- A plot of C reveals that the beam always breaks at the end points.

A combined beam and strut
Motivation

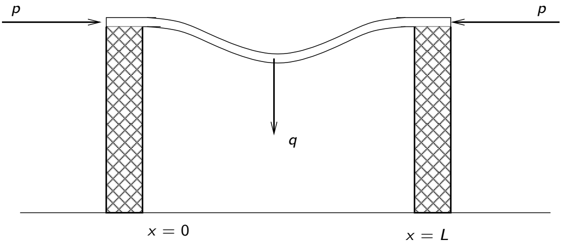


Figure: A combined beam and strut

A combined beam and strut

Governing equations

- Use the Euler-Bernoulli equation and the theory of Euler strut.
- For a unified model, we use the previous two models to have the equation

$$\frac{d^4 w}{dx^4} + \frac{p}{EI}\frac{d^2 w}{dx^2} = -\frac{q}{EI}. \tag{27}$$

- with the boundary conditions

$$\begin{aligned} w(0) &= 0, \quad \tfrac{dw}{dx}(0) = 0 \text{ at } x = 0; \\ w(L) &= 0, \quad \tfrac{dw}{dx}(L) = 0 \text{ at } x = L. \end{aligned} \tag{28}$$

- The nondimensional form of the model is

$$\frac{d^4 \bar{w}}{d\bar{x}^4} + \frac{pL^2}{EI}\frac{d^2 \bar{w}}{d\bar{x}^2} = -\frac{qL^4}{EIs}. \tag{29}$$

A combined beam and strut

Governing equations

- Define the so called *Mason number* M and s such that

$$M^2 = \frac{pL^2}{EI}, \qquad s = \frac{qL^4}{EI} \tag{30}$$

- The problem reduces to

$$\frac{d^4\bar{w}}{d\bar{x}^4} + M^2\frac{d^2\bar{w}}{d\bar{x}^2} = -1. \tag{31}$$

- with the boundary conditions

$$\begin{aligned} \bar{w}(0) = 0, \quad & \tfrac{d\bar{w}}{d\bar{x}}(0) = 0 \text{ at } x = 0; \\ \bar{w}(1) = 0, \quad & \tfrac{d\bar{w}}{d\bar{x}}(1) = 0 \text{ at } x = 1. \end{aligned} \tag{32}$$

A combined beam and strut

Solution to the Governing equations

- The solution of the previous equation is found as

$$\bar{w}(x) = \frac{1}{2M^2}\left(\frac{1}{4} - (x - \frac{1}{2})^2\right) + \frac{1}{2M^3 \sin\frac{M}{2}}\left(\cos\frac{M}{2} - \cos(M(x - \frac{1}{2}))\right) \quad (33)$$

- The curvature is mesured with the help of

$$|\bar{w}''(x)| = \left| -\frac{1}{M^2} + \frac{\cos(M(x - \frac{1}{2}))}{2M \sin\frac{M}{2}} \right| .$$

A combined beam and strut

Numerical Solution to the Governing equations

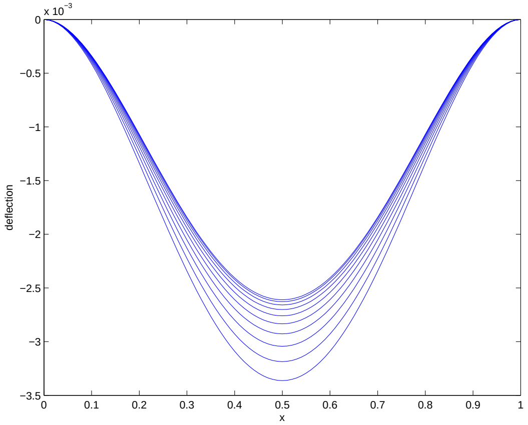


Figure: A plot of the deflection for different values of M, $M = 0.3 - 3$.

A combined beam and strut

Numerical Results

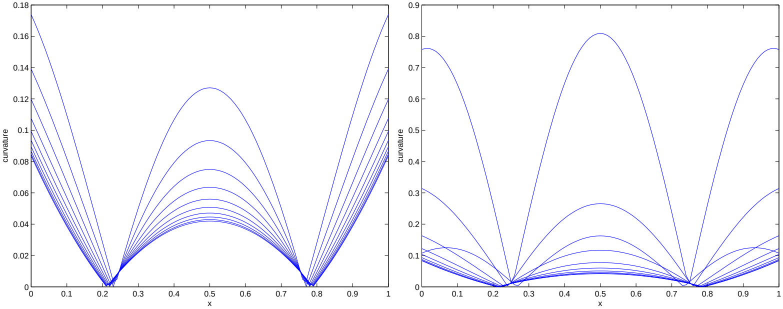


Figure: A plot of the curvature $|w''(x)|$ for different values of M.

A combined beam and strut
Numerical Results

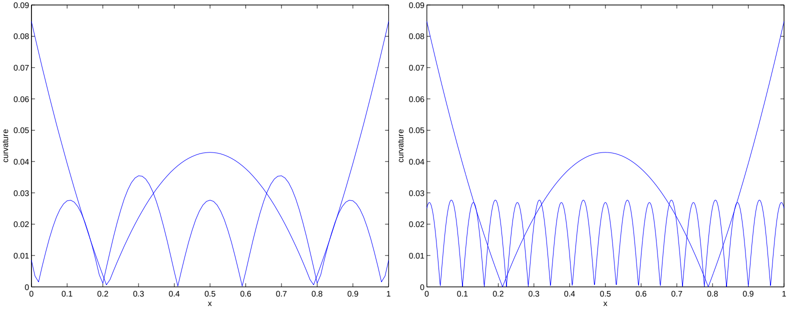


Figure: A plot of the curvature $|w''(x)|$ for different values of M.

A combined beam and strut
Numerical Results

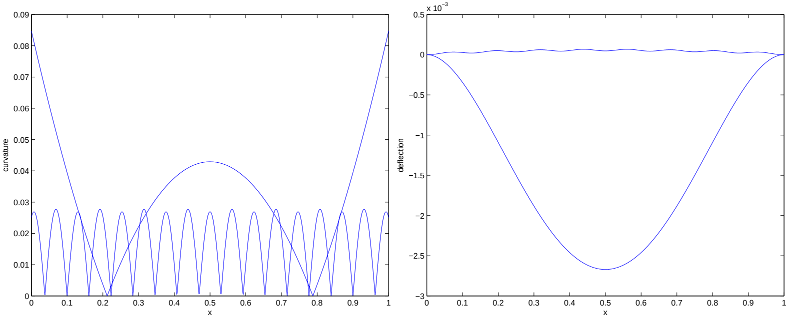


Figure: A plot of the deflection and the curvature $M = 0.8$ and $M = 8$.

A combined beam and strut

Important results

- It appear that for small $(M < 2\pi)$ values of M, the beam break at the end points.
- We never recover the first mode of the solution of the Euler strut equation.
- This is due to the fact that for the beam we have 4 boundary conditions and only 2 for the Euler strut. The equivalent fourth order ODE associated with the Euler strut has different boundary conditions.

Conclusions

- Snook collapse
  - Stress is dependent of the quantity $\frac{L}{H}$. This ration of height to width of the pillar is crucial. In general mining practices, $\frac{L}{H} \geq 5$; but for secondary mining operations, we can have $\frac{L}{H} < 5$.
  - In this later case, the stress is confined to boundary layers and the problem correspond to singularly perturbed problem.
- Roof stability
  - We found a critical ratio, M, for the roof to buckle or to break.
  - When $M < 2\pi$, what happen in most of the cases, the roof always break next to the snook.

Thank you

Thank you

THANK YOU