THE SOCCER MATCH BALL PROBLEM

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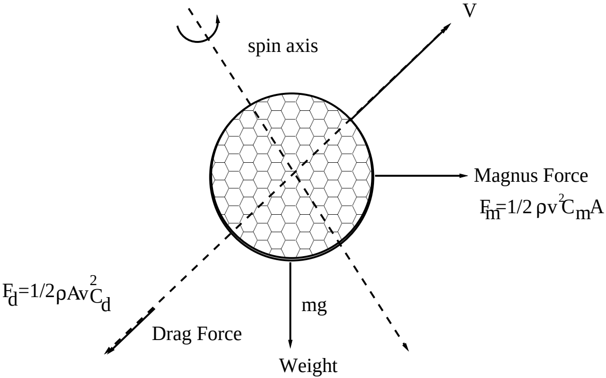
January 14, 2011


V
spin axis
Magnus Force
F_m=1/2 $\rho v^2 C_m A$
F_d=1/2$\rho A v^2 C_d$
Drag Force
mg
Weight

The Equation of motions

$$m\frac{d^2x}{dt^2} = F_D + F_L + F_S + mg \quad (1)$$

where

$$F_D = \frac{1}{2}\rho A|\dot{x}|^2 C_D\left(-\hat{\dot{x}}\right) \quad (2)$$

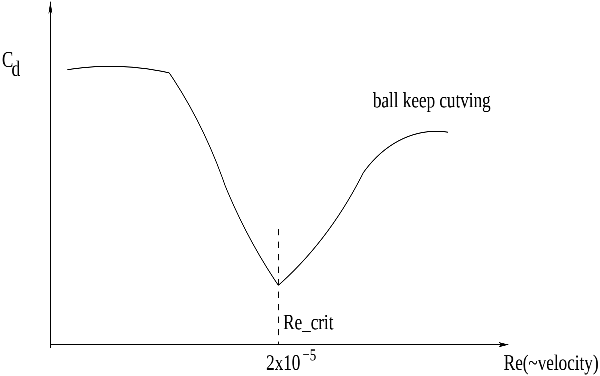
$$F_L = \frac{1}{2}\rho A|\dot{x}|^2 C_L \hat{n} \quad (3)$$

$$F_S = \frac{1}{2}\rho A|\dot{x}|^2 C_S\left(\hat{n} \times \hat{v}\right) \quad (4)$$

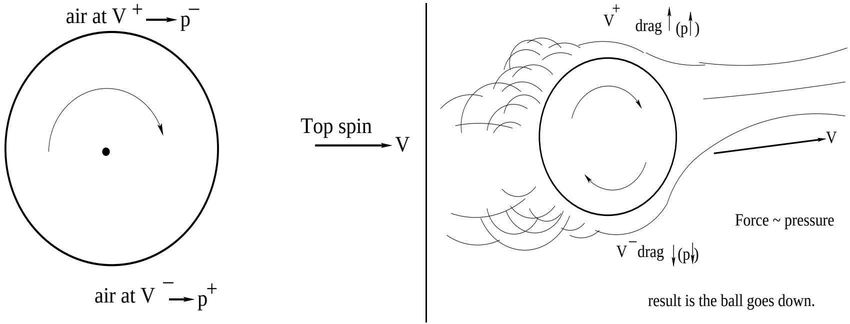
- ρ is the density of the air, a is the ball radius , ω is the angular velocity
- U is the velocity of the ball , μ is the dynamic air viscosity

If Spin is zero, $C_L = 0$ and $C_S = 0$ The coefficients C_L, C_S and C_D depend on

- $$Re = \rho \frac{Ua}{\mu} \tag{5}$$
- $$Sp = \frac{a\omega}{U} \tag{6}$$
- $$Roughness = \frac{k}{a} \tag{7}$$



C_d
ball keep cutving
Re_crit
$2x10^{-5}$
Re(~velocity)


air at V + → p −
air at V − → p +
Top spin
V
V +
drag ↑ (p↑)
V
Force ~ pressure
V − drag ↓ (p↓)
result is the ball goes down.

Ball Flight Differential Equation

$$\frac{d^2x}{dt^2} = -v\left(k_d\frac{dx}{dt} - k_l sin\gamma\frac{dy}{dt}\right)$$

$$\frac{d^2y}{dt^2} = -v\left(k_d\frac{dy}{dt} + k_l\left(cos\gamma\frac{dz}{dt} + sin\gamma\frac{dx}{dt}\right)\right)$$

$$\frac{d^2z}{dt^2} = -g - v\left(k_d\frac{dz}{dt} - k_l cos\gamma\frac{dy}{dt}\right) \tag{8}$$

$$\frac{d^2x}{dt^2} = k_l sin\gamma \left(\frac{dy}{dt}\right)^2$$

$$\frac{d^2y}{dt^2} = -k_d \left(\frac{dy}{dt}\right)^2$$

$$\frac{d^2z}{dt^2} = -g + k_l cos\gamma \left(\frac{dy}{dt}\right)^2 \tag{9}$$

with solution

$$y(t) = \frac{1}{k_d} ln(1 + \dot{y}_0 k_d t) \tag{10}$$

$$x(t) = \dot{x}_0 t - \frac{k_l}{k_d^2} sin\gamma ln(1 + \dot{y}_0 k_d t) + \frac{k_l}{k_d} sin\gamma \dot{y}_0 t \tag{11}$$

$$z(t) = \dot{z}_0 t - \frac{1}{2} g t^2 - \frac{k_l}{k_d} cos\gamma ln(1 + \dot{y}_0 k_d t) + \frac{k_l}{k_d} cos\gamma \dot{y}_0 t \tag{12}$$

By expanding $ln\left(1+k_d\dot{y_0}t\right)$ for small $k_d\dot{y}_0t$ ($k_d\sim 10^{-2}$, $\dot{y}_0\sim 20$), we have

$$ln\left(1+k_d\dot{y}_0t\right)\sim k_d\dot{y}_0t+\frac{1}{2}k_d^2\dot{y_0}^2t^2 \tag{13}$$

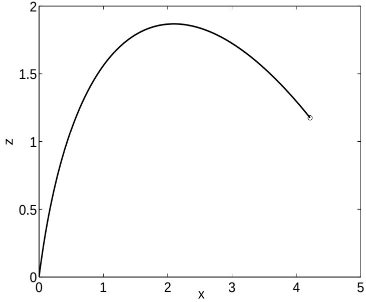
Hence, we obtain

$$y(t)=\dot{y}_0t-k_d\dot{y_0}^2\frac{t^2}{2} \tag{14}$$

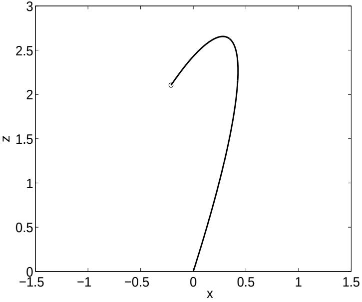
$$x(t)=\dot{x}_0t+k_l sin\gamma\dot{y_0}^2\frac{t^2}{2} \tag{15}$$

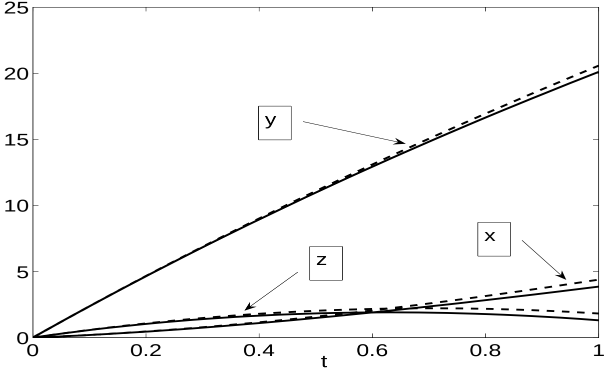
$$z(t)=\dot{z}_0t-\frac{t^2}{2}\left(g-k_l cos\gamma\dot{y_0}^2\right) \tag{16}$$

(a)



(b)





25
20
15
10
5
0
0
0.2
0.4
0.6
0.8
1
t
y
x
z

What use is this?

(or How do I use this knowledge to beat Ajax?)

$$y(t) = \dot{y}_0 t - k_d \dot{y}_0{}^2 \frac{t^2}{2} \quad (17)$$

$$x(t) = \dot{x}_0 t + k_l sin\gamma \dot{y}_0{}^2 \frac{t^2}{2} \quad (18)$$

$$z(t) = \dot{z}_0 t - \frac{t^2}{2} \left(g - k_l cos\gamma \dot{y}_0{}^2\right) \quad (19)$$

Forward motion depends on $k_d, \dot{y}_0$
Swerve depends on $k_l, sin\gamma, \dot{y}_0$

$$k_d = \frac{\rho A C_d}{2m} \qquad k_l = \frac{\rho A C_l}{2m} \quad (20)$$

$$C_d = C_d(Re, Sp, Ro) \qquad C_l = C_l(Re, Sp, Ro) \quad (21)$$

Difference in ball behaviour between cities

I can kick the ball with a certain amount of spin so how does ball behaviour differ between cities
Motion depends on Re, Ro

Ro accounts for pattern and smoothness

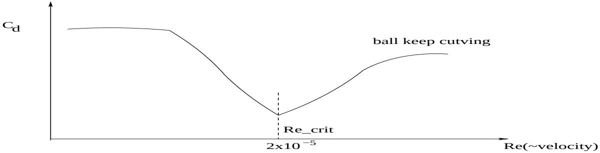
$Re = \rho U a/\mu$ – ρ varies significantly

$\rho(\text{CT}) \approx 1.29\text{kg/m}^3$ hence Re high

$\rho(\text{JB}) \approx 1.04\text{kg/m}^3$ hence Re low

Drag higher in CT - CT players expect more curve

What ball to use?



When $Re > Re_{crit}$ ball keeps on curving
Increasing Ro decreases R_{crit}
(Reverse Magnus around Re_{crit} ... Jabulani)

Playing in Jo'burg
T90 is dimpled, so curves best, hence play with smooth ball (and/or smaller ball)

Playing in CT
Practice with T90 (and/or larger ball) (and hope they choose a smooth ball)