Blood assignment problem

Blessing, Dessalegn, Eva Liliane, Jean, Mantombi, Milaine

Industry representative: Aderemi Adewumi

Friday 14 January 2011

MISGSA 2011, 10-14 January, Wits

World of blood

- Demand for blood
- Supply of blood
- Match supply and demand
- Blood groups
- Rhesus

Receiver/Donor	O	A	B	C
O	V	X	X	X
A	V	V	X	X
B	V	X	V	X
C	V	V	V	V

- Emergencies

Study group objectives

- Optimize blood resources
- Make replacement policy decisions
- Assumptions
  - No emergency
  - No rhesus
  - Need and supply proportional to population
- Develop model

In the following...

- Study mass balance
- Determine a dynamic system
- Decision making

Blood requirements

- Total blood needed
- Blood of each type

$$\begin{aligned}\Delta_O &= D_{OO} \\ \Delta_A &= D_{OA} + D_{AA} \\ \Delta_B &= D_{OB} + D_{BB} \\ \Delta_C &= D_{OC} + D_{AC} + D_{BC} + D_{CC}\end{aligned}$$

- D_{XY}: quantity of blood X used to replace blood Y

$$D_{XY} \propto V_X$$

- Importance of simplification

Parameter analysis

- Equations

$$
\begin{aligned}
\Delta_O &= \alpha_1 V_O \\
\Delta_A &= \alpha_2 V_O + \beta_2 V_A \\
\Delta_B &= \alpha_3 V_O \qquad\qquad + \gamma_3 V_B \\
\Delta_C &= \alpha_4 V_O + \beta_4 V_A + \gamma_4 V_B + \delta_4 V_C
\end{aligned}
$$

- 9 parameters
- 4 equations
- Choose 5 parameters

Dynamical mass balance

- Blood conservation

$$
\begin{aligned}
\frac{d\,V_O}{dt} &= Q_O - (D_{OO} + D_{OA} + D_{OB} + D_{OC}) \\
\frac{d\,V_A}{dt} &= Q_A - (D_{AA} + D_{AC}) \\
\frac{d\,V_B}{dt} &= Q_B - (D_{BB} + D_{BC}) \\
\frac{d\,V_C}{dt} &= Q_C - (D_{CC})
\end{aligned}
$$

- Blood collected
- Blood used

Differential equations

- Governing equations

$$
\begin{aligned}
\frac{d\,V_O}{dt} &= Q_O - (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4)\,V_O \\
\frac{d\,V_A}{dt} &= Q_A - (\qquad \beta_2 + \qquad \beta_4)\,V_A \\
\frac{d\,V_B}{dt} &= Q_B - (\qquad\qquad \gamma_3 + \gamma_4)\,V_B \\
\frac{d\,V_C}{dt} &= Q_C - (\qquad\qquad\qquad \delta_4)\,V_C
\end{aligned}
$$

- Standard ODEs
- Explicit expressions $V_O(t)$, $V_A(t)$, $V_B(t)$, $V_C(t)$
- Parameters still unknown

Blood bank management

- Ideal situation

$$(n_O,\ n_A,\ n_B,\ n_C)$$

- Manage resources to reach goal
- Actual proportions

$$(p_O,\ p_A,\ p_B,\ p_C)$$

- Expression at the end of the day

$$p_O = \frac{V_O(1)}{V_O(1) + V_A(1) + V_B(1) + V_C(1)}$$

Objective function

- Optimise values of parameters
- Objective function

$$E = (p_O - n_O)^2 + (p_A - n_A)^2 + (p_B - n_B)^2 + (p_C - n_C)^2$$

- Possible weights for components
- Choice dependent of blood shortages

$$\begin{aligned} E &= \epsilon_1 (p_O - n_O)^2 + \epsilon_2 (p_A - n_A)^2 \\ &\quad + \epsilon_3 (p_B - n_B)^2 + \epsilon_4 (p_C - n_C)^2 \end{aligned}$$

Optimisation method

- Minimise Objective function
- Possible solutions
  - Gradient
  - Other standard methods
  - Numerical solution
- Decision making

$$r_{OA} = \frac{D_{OA}}{\Delta_A}$$

Conclusion

- What was achieved:
  - Model developed
    - Dynamic system
    - Linear equations
  - Decision making
    - Objective function defined
    - Choice optimisation parameters

Future work

- What should be done
  - Include
    - Rhesus
    - Emergencies
    - Blood components
    - Life span of products
  - Analyse more data
  - Numerical results