Lane-Emdem Equation of the Density Distribution of an Isothermal Gas Sphere

Graduate Student Workshop MISG 2011 @ WITS

January 8, 2011

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Ludovic, Siboleke, Eva & Dennis Lane-Emden equation

Outline

- Team Members
 - Team Members
- 2 Introduction
 - Density Model Formulation
 - Lane-Emden Equation
- Solution of The Lane Emden Equation
 - Literature Survey
 - Adomian Decomposition
 - Finite Difference Scheme
- Analysis of Solution
 - Findings
 - Analysis

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Team Members

Introduction Solution of The Lane Emden Equation Analysis of Solution

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Team Members

Introduction Solution of The Lane Emden Equation Analysis of Solution

Team Members

Team Members

Who we are!

- Harley Charis WITS (Supervisor)
- Tangpi Ludovic AIMS
- Mzwana Siboleke WITS
- Ujeneza Eva-Liliane AIMS
- Ikpe Dennis UNISA

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Density Model Formulation Lane-Emden Equation

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Density Model Formulation Lane-Emden Equation

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Density Model Formulation

Modelling the density of an isothermal gas sphere



Figure: Gas ball in hydrostatic equilibrium

Density Model Formulation Lane-Emden Equation

Density Model Formulation

• Given the poisson equation:

$$\frac{dM}{dr} = 4\pi\rho(r)r^2 \tag{1}$$

and the equation of hydrostatic equilibrium:

$$\frac{dP}{dr} = \frac{M(r)\rho(r)G}{r^2}$$
(2)

• By combining , (1) and (2) we have

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho(r)}\frac{dP}{dr}\right) = -4\pi G\rho \tag{3}$$

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Density Model Formulation Lane-Emden Equation

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Density Model Formulation Lane-Emden Equation

Lane-Emden Equation Of an Isothermal Gas Sphere

 Using the thermodynamic properties of an Isothemal gas sphere, we have

$$P = K\rho + D \tag{4}$$

Where K and D are constants.

• Two important tranformations:

$$\rho = \rho_c \boldsymbol{e}^{-\boldsymbol{y}}, \, \boldsymbol{r} = \left[\frac{\kappa}{4\pi G \rho_c}\right]^{\frac{1}{2}} \boldsymbol{x}$$
 (5)

• Applying the transformations, we have

Lane-Emden Equation

$$y''+\frac{2}{x}y'+e^y=0$$

Literature Survey Adomian Decomposition Finite Difference Scheme

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Literature Survey Adomian Decomposition Finite Difference Scheme

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Existing Aproaches

- Adomian Decomposition (Analytical)
- Finite Difference (Numerical)

Literature Survey Adomian Decomposition Finite Difference Scheme

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Adomian Decomposition

Decomposition:

$$y(x)=\sum_{n=0}^{\infty}y_n(x)$$

$$e^{y} = \sum_{n=0}^{\infty} A_n(y_1, ... y_n)$$

Taylor Expansion of e^y:

$$\boldsymbol{e}^{\boldsymbol{y}} = \boldsymbol{e}^{\sum_{n=0}^{\infty} y_n(\boldsymbol{x})}$$

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Grouping the terms correctly gives the kth Adomian polynomial, A_k

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Finite Difference

• Forward Finite Difference.

$$y'' = \frac{y[i+1] - 2y[i] + y[i-1]}{h^2}$$
$$y' = \frac{y[i+1] - y[i]}{h}$$

FD solution

$$y[i+1] = \frac{1}{\left(1 + \frac{2h}{x[i]}\right)} \left[\frac{2h}{x[i]}y[i] - h^2 e^{y[i]} + 2y[i] - y[i-1]\right]$$

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Finite Difference



Figure: Solution the Finite Difference Scheme

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Findings Analysis

Adomian of Order 3



Figure: Solution Using Adomian decomposition of Order 3

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Findings Analysis

Adomian of Order 4



Figure: Solution Using Adomian decomposition of Order 4

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Findings Analysis

Adomian of Order 6



Figure: Solution Using Adomian decomposition of Order 6

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Findings Analysis

Adomian of Order 4 and 6



Figure: Comparison of Adomian decomposition of Order 4 and 6

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Findings Analysis

Adomian of Order 4 and 6



Figure: Comparison of Adomian decomposition of Order_4 and 6

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Findings Analysis

Adomian of Order 4 and 6



Figure: Error of Adomian decomposition of Order 4 and 6

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Findings Analysis

Density Distribution

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Findings Analysis

Density Distribution



Figure: Density distribution Adomian of Order 6

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Findings Analysis

Conclusion

- Increasing the order of The Adomian Polynomial does not significantly improve the Accuracy.
- The Lane-Emden equation Acuratly Models the density distribution of an isothermal gas sphere.

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Findings Analysis

Questions

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QUESTIONS/COMMENTS?

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