Lane-Emdem Equation of the Density Distribution of an Isothermal Gas Sphere

Graduate Student Workshop
MISG 2011 @ WITS

January 8, 2011

Outline

1. Team Members
   - Team Members
2. Introduction
   - Density Model Formulation
   - Lane-Emden Equation
3. Solution of The Lane Emden Equation
   - Literature Survey
   - Adomian Decomposition
   - Finite Difference Scheme
4. Analysis of Solution
   - Findings
   - Analysis

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Team Members

Who we are!

- Harley Charis - WITS (Supervisor)
- Tangpi Ludovic - AIMS
- Mzwana Siboleke - WITS
- Ujeneza Eva-Liliane - AIMS
- Ikpe Dennis - UNISA

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Density Model Formulation

- Modelling the density of an isothermal gas sphere

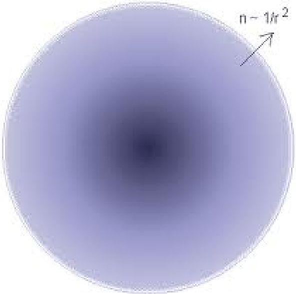

Figure: Gas ball in hydrostatic equilibrium

Density Model Formulation

- Given the poisson equation:

$$\frac{dM}{dr} = 4\pi\rho(r)r^2 \tag{1}$$

and the equation of hydrostatic equilibrium:

$$\frac{dP}{dr} = \frac{M(r)\rho(r)G}{r^2} \tag{2}$$

- By combining , (1) and (2) we have

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho(r)}\frac{dP}{dr}\right) = -4\pi G\rho \tag{3}$$

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Lane-Emden Equation Of an Isothermal Gas Sphere

- Using the thermodynamic properties of an Isothemal gas sphere, we have

$$P = K\rho + D \tag{4}$$

Where K and D are constants.

- Two important tranformations:

$$\rho = \rho_c e^{-y}\ ,\ r = \left[\frac{K}{4\pi G\rho_c}\right]^{\frac{1}{2}} x \tag{5}$$

- Applying the transformations, we have

Lane-Emden Equation

$$y'' + \frac{2}{x}y' + e^{y} = 0$$

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Existing Aproaches

- Adomian Decomposition (Analytical)
- Finite Difference (Numerical)

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Adomian Decomposition

- Decomposition:

$$y(x) = \sum_{n=0}^{\infty} y_n(x)$$

$$e^y = \sum_{n=0}^{\infty} A_n(y_1, ...y_n)$$

- Taylor Expansion of e^y:

$$e^y = e^{\sum_{n=0}^{\infty} y_n(x)}$$

- Grouping the terms correctly gives the kth Adomian polynomial, A_k

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Finite Difference

- Forward Finite Difference.

$$y'' = \frac{y[i+1] - 2y[i] + y[i-1]}{h^2}$$

$$y' = \frac{y[i+1] - y[i]}{h}$$

- FD solution

$$y[i+1] = \frac{1}{\left(1 + \frac{2h}{x[i]}\right)} \left[\frac{2h}{x[i]} y[i] - h^2 e^{y[i]} + 2y[i] - y[i-1] \right]$$

Finite Difference

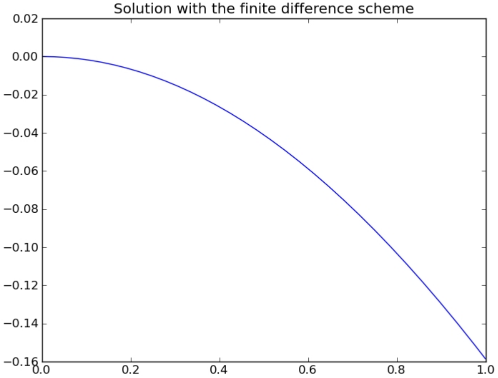

Figure: Solution the Finite Difference Scheme

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Adomian of Order 3

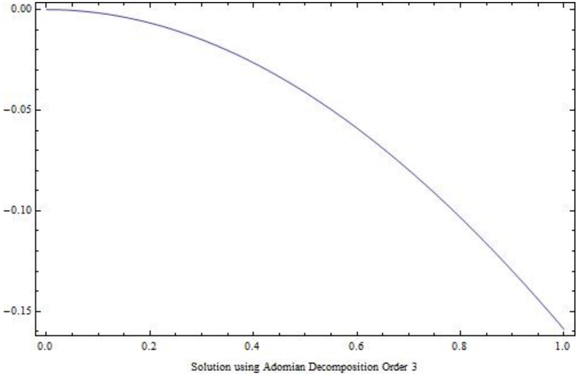

Figure: Solution Using Adomian decomposition of Order 3

Adomian of Order 4

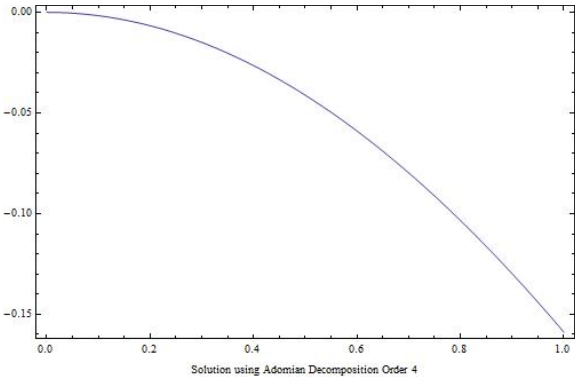

Figure: Solution Using Adomian decomposition of Order 4

Adomian of Order 6

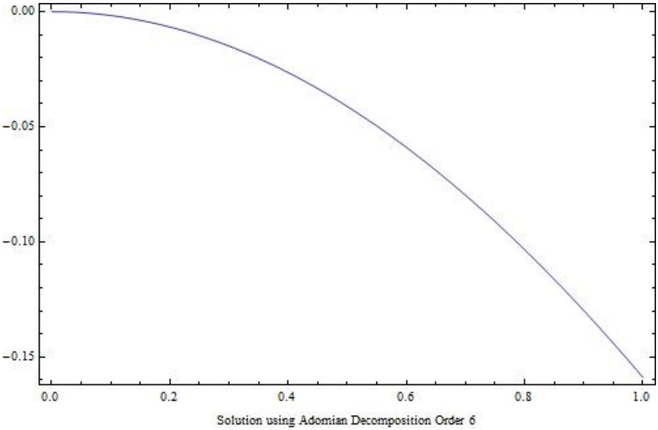

Figure: Solution Using Adomian decomposition of Order 6

Adomian of Order 4 and 6

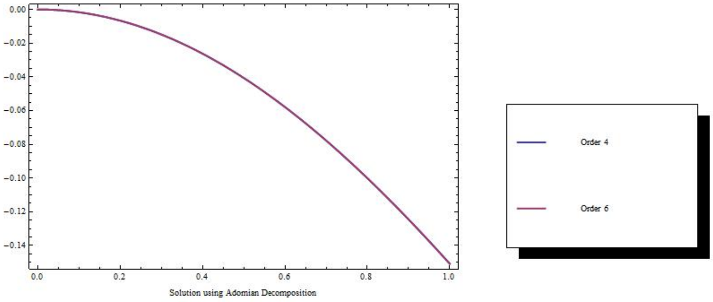

Figure: Comparison of Adomian decomposition of Order 4 and 6

Adomian of Order 4 and 6

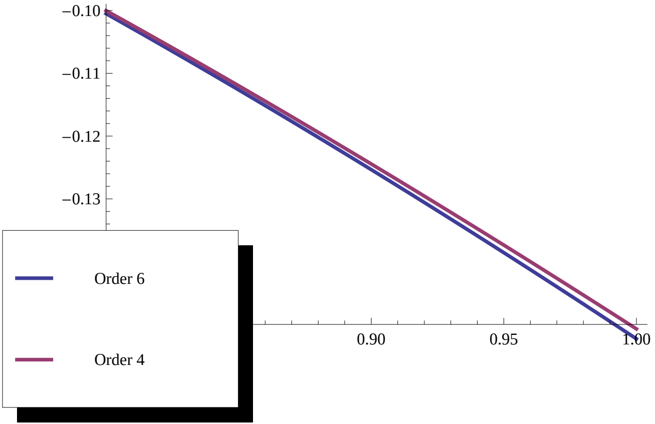

Figure: Comparison of Adomian decomposition of Order 4 and 6

Adomian of Order 4 and 6

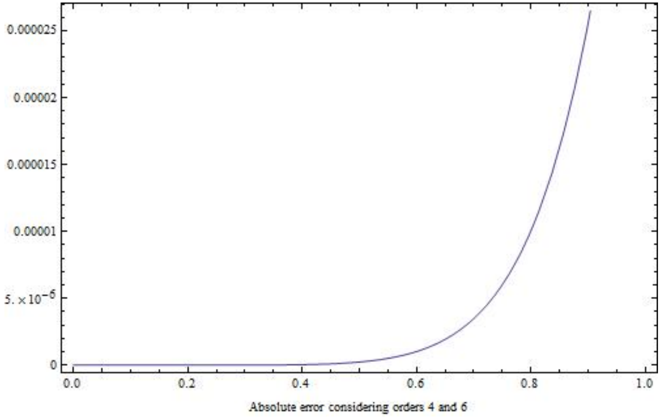

Figure: Error of Adomian decomposition of Order 4 and 6

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Density Distribution

Density Distribution

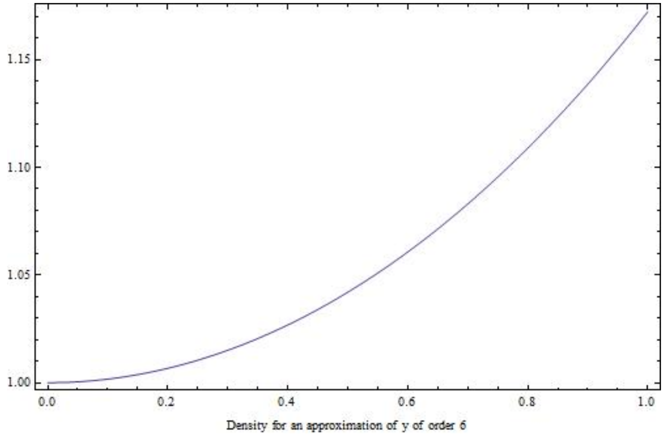

Figure: Density distribution Adomian of Order 6

Conclusion

- Increasing the order of The Adomian Polynomial does not significantly improve the Accuracy.
- The Lane-Emden equation Acuratly Models the density distribution of an isothermal gas sphere.

Questions

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QUESTIONS/COMMENTS?