HYDRAULIC FRACTURE

Hydraulic Fracture with Leak off

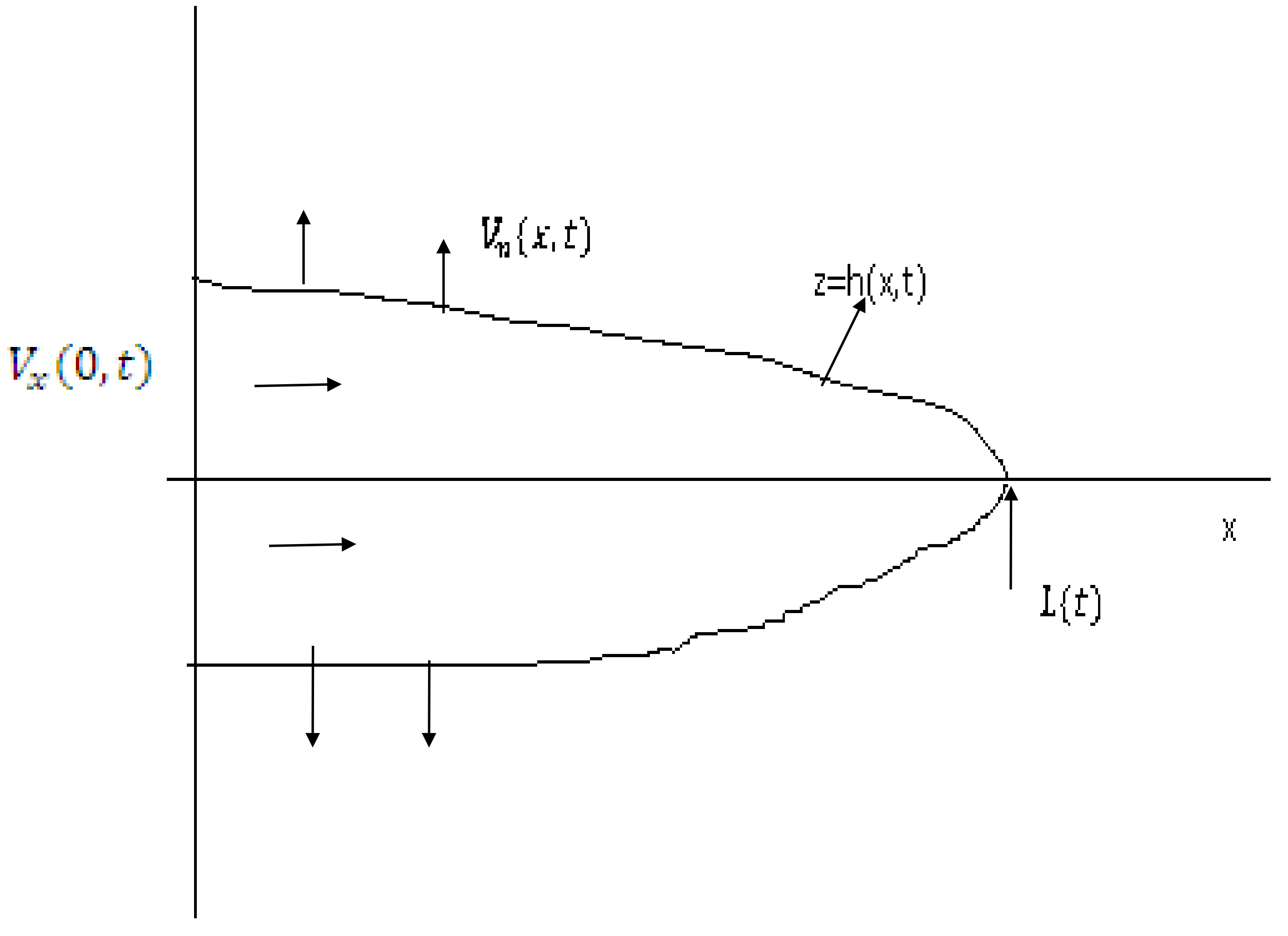
INTRODUCTION

- Hydraulic fracturing is a process that is used to create fractures in rocks.
- It was first used in the US in 1947, and went commercial in 1949.
- Its success in increasing production from oil wells made it to be adapted worldwide.
- The most important industrial use is in stimulating oil and gas wells.

- Investigate effect of leak-off on growth of fracture
- Investigate the extraction of fluid from fracture.

Hydraulic Fracture with Leak off

- The fluid is pumped into the fracture by a fluid injection at a velocity v_x
- The cavity walls are permeable so some amount of fluid escapes into or its sucked out of the permeable rock at a leak off velocity $v_n(x,t)$
- $h(x,t) = \frac{1}{2} \times$ width of fracture.



$V_n(x,t)$
z=h(x,t)
$V_x(0,t)$
x
$L(t)$

- The PDE that describes the situation mathematically is given by:

$$\frac{\partial h(x,t)}{\partial t} = \frac{\Lambda}{3\mu}\frac{\partial}{\partial x}\left(h^3\frac{\partial h(x,t)}{\partial x}\right) - v_n(x,t)$$

- Firstly, we derive the above equation
- Then the above PDE was solved analytically using scaling transformation, similarity solutions.

CHARACTERISTIC EQUATIONS

$$\bar{V}_x = \frac{V_x}{U}\ , \qquad \bar{p} = \frac{p}{P}\ , \qquad P = \frac{\mu L U}{H^2}$$

$$\bar{V}_z = \frac{V_z}{W}\ , \qquad Re = \frac{UL}{\nu}\ , \qquad \nu = \frac{\mu}{p}$$

BY SUBSTITUTING THE ABOVE SCALAR EQUATIONS INTO EULERS EQUATION WE GET

$$Re\left(\frac{H}{L}\right)^2\left[\frac{\partial \bar{V}_x}{\partial t} + \bar{V}_x\ \frac{\partial \bar{V}_x}{\partial \bar{x}} + \bar{V}_z\ \frac{\partial \bar{V}_x}{\partial \bar{z}}\right] = -\frac{\partial p}{\partial \bar{x}} + \left(\frac{H}{L}\right)^2 \frac{\partial \bar{V}_x}{\partial \overline{x^2}} + \frac{\partial^2 \bar{V}_x}{\partial \bar{z}} \ldots \ldots . . (1)$$

$$Re\left(\frac{H}{L}\right)^2\left[\frac{\partial \bar{V}_z}{\partial t} + \bar{V}_x\ \frac{\partial \bar{V}_z}{\partial \bar{x}} + \bar{V}_z\ \frac{\partial \bar{V}_z}{\partial \bar{z}}\right] = -\frac{\partial p}{\partial \bar{z}} + \left(\frac{H}{L}\right)^4 \frac{\partial \bar{V}_z}{\partial \overline{x^2}} + \left(\frac{H}{L}\right)^2 \frac{\partial^2 \bar{V}_z}{\partial \bar{z}} \ldots \ldots . . (2)$$

CONTINUITY EQUATION

$$\frac{\partial \bar{V}_x}{\partial \bar{x}} + \frac{\partial \bar{V}_Z}{\partial \bar{z}} = 0 \ldots \ldots \ldots (3)$$

LUBRICATION APPROXIMATION

$$\frac{H}{L} \ll 1, \quad Re\left(\frac{H}{L}\right)^2 \ll 1$$

$$\frac{\partial^2 \bar{V}_x}{\partial \bar{z}^2} = \frac{\partial \bar{p}}{\partial \bar{x}}$$

$$\frac{\partial \bar{p}}{\partial \bar{z}} = 0$$

$$\frac{\partial \bar{V}_x}{\partial \bar{x}} + \frac{\partial \bar{V}_Z}{\partial \bar{z}} = 0$$

Boundary Conditions

$$z = h(x,t) \qquad V_x(x,h,t) = 0$$

$$z = h(x,t) \qquad V_Z(x,h,t) = \frac{\partial h}{\partial t} + V_n(x,t)$$

$$z = -h(x,t) \qquad V_x(x,-h,t) = 0$$

$$z = -h(x,t) \qquad V_z(x,-h,t) = -\frac{\partial h}{\partial t} - V_n(x,t)$$

Using the continuity equation (3), we first showed that

$$\frac{\partial h}{\partial t} + \frac{1}{2}\frac{\partial}{\partial x}\int_{-h(x,t)}^{h(x,t)} V_x(x,z,t)dz = -V_n(x,t) \text{..........(4)}$$

We solved for $V_x(x,z,t)$

$$V_x(x,z,t) = \frac{1}{2\mu}\frac{\partial p(x,t)}{\partial x}(z^2 - h^2)$$

Substituting into (4),

$$\frac{\partial h}{\partial t} = \frac{1}{3\mu}\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right) - V_n(x,t)$$

Using simple model, we took $p(x,t) = \Lambda h(x,t)$

Thus we have our equation

$$\boldsymbol{\frac{\partial h}{\partial t} = \frac{\Lambda}{3\mu}\frac{\partial}{\partial x}\left[h^3\frac{\partial h}{\partial x}\right] - V_n(x,t)}$$

PDE

$$\frac{\partial h}{\partial t} = \frac{\lambda}{3\mu} \frac{\partial}{\partial x} \left[h^3 \frac{\partial h}{\partial x} \right] - V_n(x,t)$$

$WE\ USE\ THE\ FOLLOWING\ FUNCTIONS$:

$h(x,t) = t^{\frac{1}{3}(2\propto-1)} F(\xi)$

$V_n(x,t) = t^{\frac{1}{3}(\propto-2)} G(\xi)$

$WHERE: \xi = \dfrac{x}{t^{\propto}}$

$$\frac{\lambda}{3\mu} \frac{d}{d\xi} \left[F^3(\xi) \frac{dF(\xi)}{d\xi} \right] + \propto \frac{d}{d\xi} (\xi F) - \frac{5}{3} \left(\propto - \frac{1}{5} \right) F(\xi) - G(\xi)$$

BY USING

$$H(\eta) = AF(\xi), \quad F(\xi) = \frac{1}{A}H(\eta)$$

$$W(\eta) = BG(\xi), \quad G(\xi) = \frac{1}{B}W(\eta)$$

$$WHERE\colon \eta = \frac{\xi}{L_0} = \frac{x}{L_0 t^{\propto}} = \frac{x}{L(t)}$$

$$h(x,t) = t^{\frac{1}{3}(2\propto -1)} \left(\frac{3\mu L_0^2}{\lambda}\right)^3 H(\eta)$$

$WE\ END\ UP\ WITH$

$$\frac{d}{d\eta}\left[H^3 \frac{dH}{d\eta}\right] + \propto \frac{d}{d\eta}(\eta H) - \frac{5}{3}\left(\propto - \frac{1}{5}\right) - W = 0$$

We now have the following ODE

$$\frac{d}{d\gamma}[H^3\ \frac{d}{d\gamma}H] + \alpha\frac{d}{d\gamma}[\gamma H] - (\frac{5}{3})(\alpha - \frac{1}{5}))H - W = 0$$

with boundary condition **H(1) = 0**

<u>Assumption</u>: **W= ßH**

$$\frac{d}{d\gamma}[H^3\ \frac{d}{d\gamma}H] + \alpha\frac{d}{d\gamma}[\gamma H] - (\frac{5}{3})(\alpha - \frac{1}{5}))H - \text{ßH} = 0$$

Case 1: $ß = \frac{5}{3}(\frac{1}{5} - \alpha)$

Substituting in we obtain:

$$\frac{d}{d\gamma}[H^3 \frac{d}{d\gamma}H] + \alpha\frac{d}{d\gamma}[\gamma H] = 0$$, H(1) = 0

solving this we obtained the following results:

$$H(\gamma) = [\frac{3}{2}\alpha(1 - \gamma^2)]\frac{1}{3}$$

$$h(x,t) = t^{\frac{1}{3}(2\alpha-1)}\left(\frac{\Lambda L_o{}^2}{3\mu}\right)^{\frac{1}{3}} H\left(\frac{x}{L_o t^{\alpha}}\right)$$

Interpretation:

volume in cavity $V(t) = 2\int_0^{L(t)} h(t,x)\,dx$

$$\frac{dV}{dt} = \int_{-h}^{h} V_x\ (0,t)dz - 2\int_0^{L(t)} V_n(t,x)\,dx$$

<u>Constant volume</u> : α= $\frac{1}{5}$

<u>dV/dt constant</u>: α = $\frac{4}{5}$

<u>Constant pressure at x = 0</u>: α = $\frac{1}{2}$

Case 2:

We left the ODE as is and made the ansatz

$$H(\gamma) = A[1 - \gamma)]^n$$

Substituting in, we obtained

$n = \frac{1}{3}$ $\quad\quad$ A = $(3\alpha)^{1/3}$

Solving for h we obtained

$$h(x,t) = t^{\frac{1}{3}(2\alpha-1)} \left(\frac{\Lambda {L_o}^2}{3\mu}\right)^{\frac{1}{3}} H\left(\frac{x}{L_o t^{\alpha}}\right)$$

The relationship between α and ß can be expressed as

$$\alpha = 1 - 3ß$$

For ß = 0 , $L(t) = L_0 t$ where $L(t)$ is the length of the cavity and L_0 is some constant.

Conditions on α and ß for inflow and outflow at the entrance of the cavity:

Q_1 = $$\int_{-h(0,t)}^{h(0,t)} V_x \; (0, z, t) dz$$

Q_2 = $$2 \int_0^{L(t)} V_n(t, x)\, dx$$

V(t) = $$2 \int_0^{L(t)} h(t, x)\, dx$$

$$\frac{dV}{dt} + Q2 = Q1$$

Results:

$$ß + \frac{5}{3}\left(\alpha - \frac{1}{5}\right) = 0$$

no injection of fluid

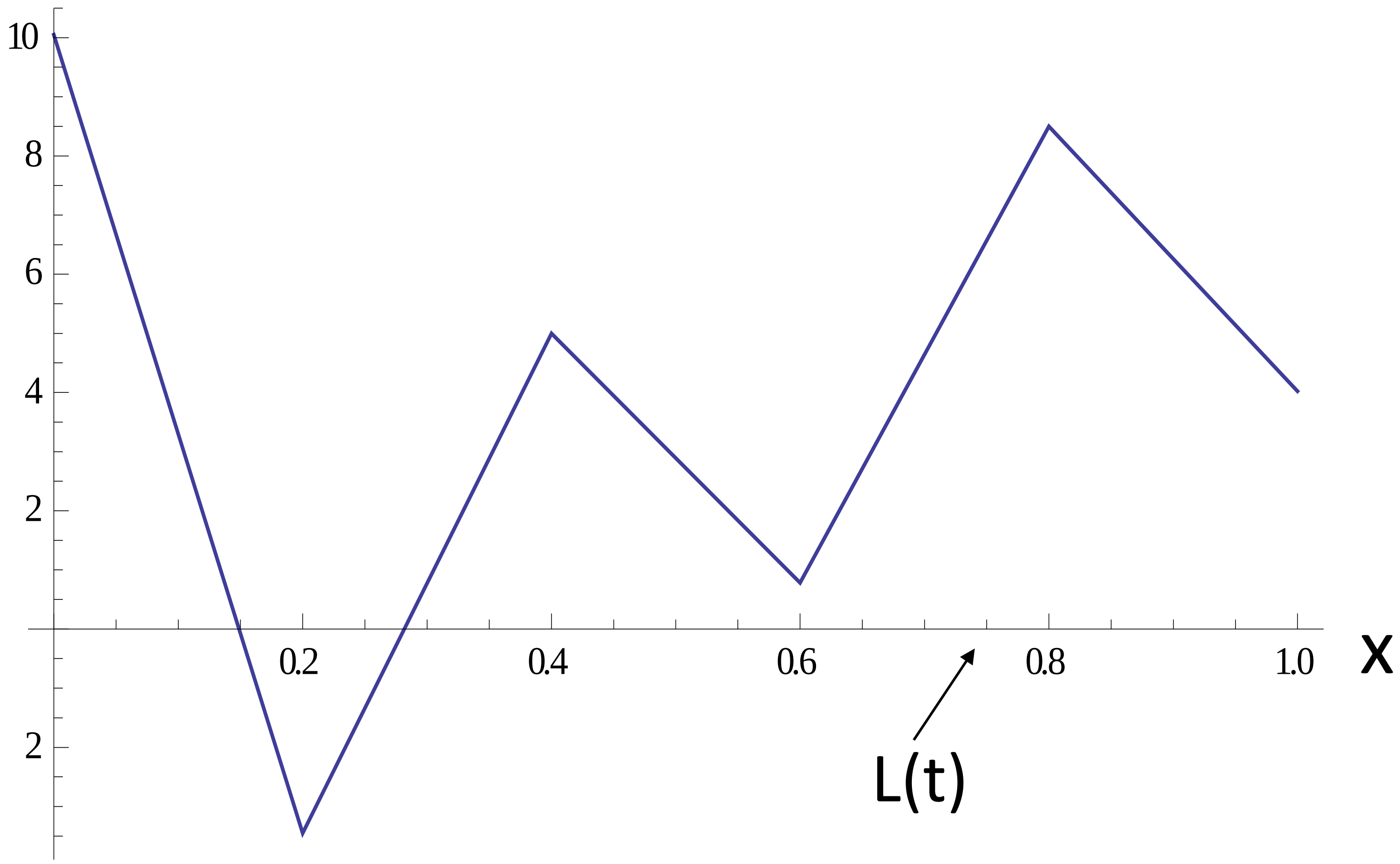
$$ß + \frac{5}{3}\left(\alpha - \frac{1}{5}\right) > 0$$

fluid is injected

$$ß + \frac{5}{3}\left(\alpha - \frac{1}{5}\right) < 0$$

fluid is extracted

A noisy solution due to the discontinuity



Finite Difference Method

$n\ =\ number\ of\ nodal\ points\ along\ \gamma\ axis$

$\Delta\ =\ step\ length$

$H_i \approx H(\gamma_i)$

$$\frac{dH}{d\gamma}(\gamma_i)\ =\ \frac{-H_{i-1}+H_{i+1}}{\Delta} +\ O(\Delta)$$

$$\frac{d^2H}{d\gamma^2}(\gamma_i)\ =\ \frac{H_{i-1}-2H_i+H_{i+1}}{\Delta} +\ O(\Delta^2)$$

Method of Lines

$$\frac{\partial h}{\partial t} = \frac{\Lambda}{3\mu}\frac{\partial}{\partial x}\left[h^3\frac{\partial h}{\partial x}\right]$$

$n\ =\ number\ of\ nodal\ points\ along\ \gamma\ axis$

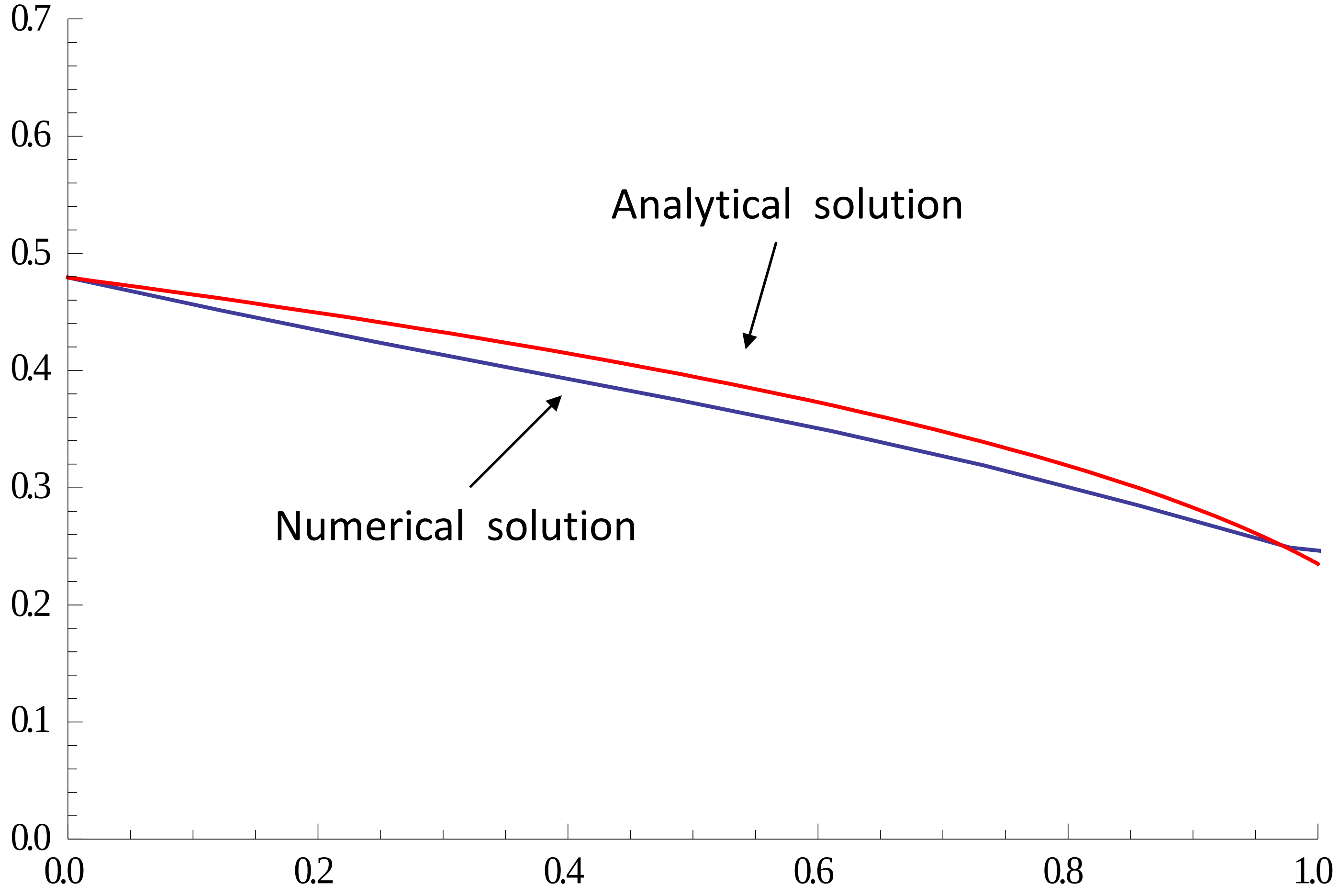
$\Delta\ =\ step\ length$

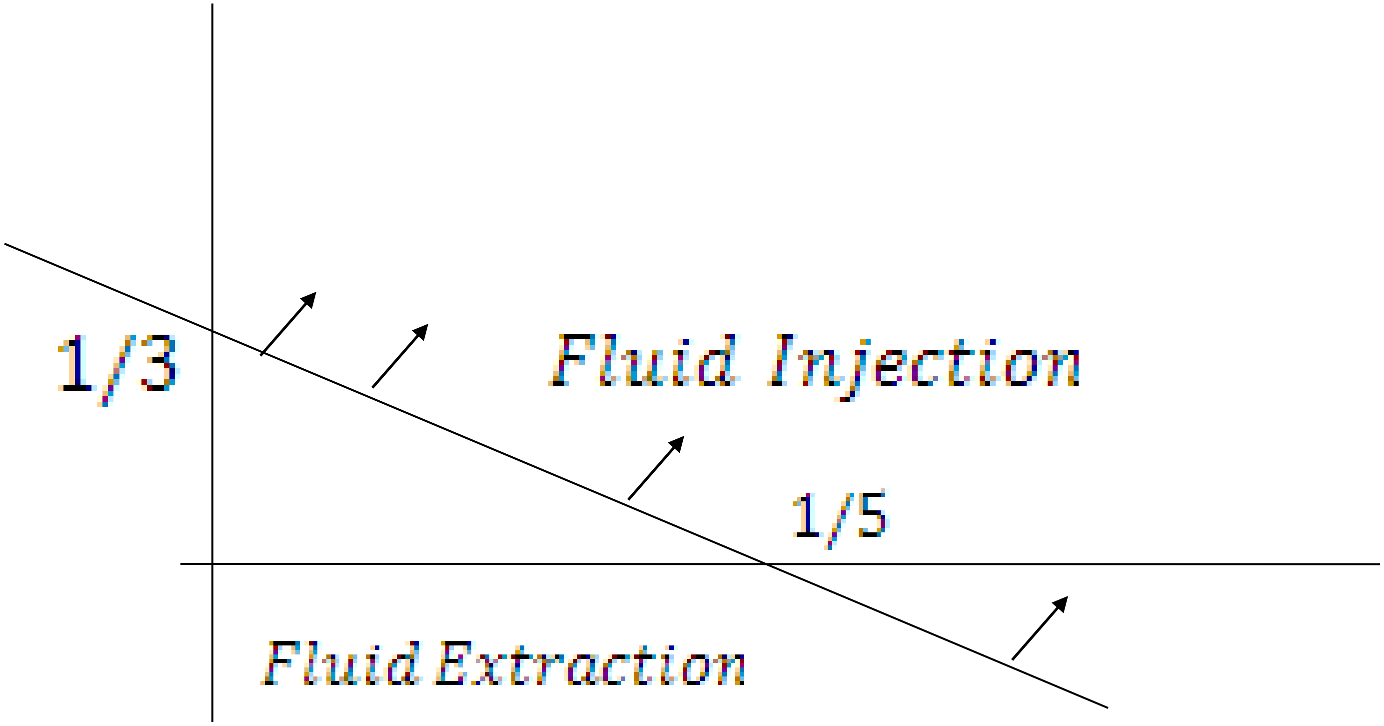
$h_i(t) \approx h(x_i, t)$

$$\frac{\partial h}{\partial x}(t, x_i) = \frac{-h_{i-1}(t) + h_{i+1}(t)}{\Delta} + O(\Delta)$$

$$\frac{\partial^2 h}{\partial x^2}(t, x_i) = \frac{h_{i-1}(t) - 2h_i(t) + h_{i+1}(t)}{\Delta} + O(\Delta^2)$$

- Plot of numerical and analytical solutions




1/3
Fluid Injection
1/5
Fluid Extraction